# Rate and equilibrium constants of $\mathrm{O}_{2}$-binding and $\mathrm{O}_{2}$ release: "The forward and reverse steps for the ${ }^{T}$ state $\rightarrow{ }^{\mathrm{R}}$ state change for human $\mathrm{Hb}_{4}$ / BPG, under standard conditions" 

Francis Knowles*, Samantha Doyle, Douglas Magde<br>Department of Chemistry and Biochemistry, University of California, San Diego, USA

## APPENDIX

Consecutive equivalent first order reactions
Consider a scheme of two consecutive pseudo-first-order reactions with stoichiometry given by
$\mathrm{Hb}_{2}+\mathrm{O}_{2} \xrightarrow[\text { excess } \mathrm{O}_{2}]{\mathrm{k}_{1}}\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}$
$\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}+\mathrm{O}_{2} \xrightarrow[\text { excess } \mathrm{O}_{2}]{\mathrm{k}_{2}}\left(\mathrm{HbO}_{2}\right)_{2}$
The hypothetical macromolecule, $\mathrm{Hb}_{2}$, is defined to be dimeric, each constituent monomer containing a heme moiety capable of reversibly binding a molecule of either $\mathrm{O}_{2}$ or CO . The dimeric model is intended to be a simplified model for human hemoglobin. In the presence of excess $\mathrm{O}_{2},\left(\mathrm{HbO}_{2}\right)(\mathrm{Hb})$ is produced as an unstable intermediate in the overall reaction given by
$\mathrm{Hb}_{2}+2 \mathrm{O}_{2} \rightarrow\left(\mathrm{HbO}_{2}\right)_{2}$
We can write the following rate equations, understanding that the rate constants, k 1 and k 2 , are pseudo first-order rate constants, the concentration of $\mathrm{O}_{2}$ being much greater than the concentration of $\mathrm{Hb}_{2}$.
$\frac{d\left[\mathrm{Hb}_{2}\right]}{d t}=-k_{1}\left[\mathrm{Hb}_{2}\right] \equiv-k_{1}^{\prime}\left[\mathrm{O}_{2}\right]\left[\mathrm{Hb}_{2}\right]$
$\frac{\left.d\left[\left(\mathrm{HbO}_{2}\right)\right) \mathrm{Hb}\right]}{d t}=k_{1}\left[\mathrm{Hb}_{2}\right]-k_{2}\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right] \equiv k_{1}^{\prime}\left[\mathrm{O}_{2}\right]\left[\mathrm{Hb}_{2}\right]-k_{2}^{\prime}\left[\mathrm{O}_{2}\right]\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]$
$\frac{d\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]}{d t}=k_{2}\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right] \equiv k_{2}^{\prime}\left[\mathrm{O}_{2}\right]\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]$
The first of these three equations is solved by the method for a firstorder equation. The equation of state for $\left[\mathrm{Hb}_{2}\right]$ is
$\left[\mathrm{Hb}_{2}\right]=\left[\mathrm{Hb}_{2}\right]_{0} \exp \left(-k_{1} t\right)$

Substituting the value for $\left[\mathrm{Hb}_{2}\right]$ into the second rate equation and rearranging, we obtain a first order linear differential equation.
$\frac{d\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]}{d t}+k_{2}\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]=k_{1}\left[\left(\mathrm{Hb}_{2}\right]_{0} \exp \left(-k_{1} t\right)\right.$
If $\left.\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right)\right] 0=0$, the solution for $\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]$ is
$\left.\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right)\right]=\left[\mathrm{Hb}_{2}\right]_{0}\left(\left(\frac{k_{1}}{k_{2}-k_{1}}\right)\left(\exp \left(-k_{1} t\right)-\exp \left(-k_{2} t\right)\right)\right)$
The solution for $\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]$ can be obtained directly from the mass conservation equations.
$\left[\mathrm{Hb}_{2}\right]_{0}=\left[\mathrm{Hb}_{2}\right]+\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]+\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]$
$\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]=\left[\mathrm{Hb}_{2}\right]_{0}-\left[\mathrm{Hb}_{2}\right]-\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]$
Substituting the equations of state for $\left[\mathrm{Hb}_{2}\right]$ and $\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]$ into the second mass conservation equation one obtains the equation of state for $\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]$.
$\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]=\left[\mathrm{Hb}_{2}\right]_{0}\left(1-\exp \left(-k_{1} t\right)-\left(\frac{k_{1}}{k_{2}-k_{1}}\right)\left(\exp \left(-k_{1} t\right)-\exp \left(-k_{2} t\right)\right)\right)$
With the equations of state presented above, it is possible to simulate data for each of the concentrations of the reactants in the conversion of $\mathrm{Hb}_{2}$ to $\left(\mathrm{HbO}_{2}\right)_{2}$.
Special considerations for models of human hemoglobin
In the case of reaction of a dimeric hemoglobin molecule with, for example, $\mathrm{O}_{2}$, it would be normal to start with a solution free of $\mathrm{O}_{2}$. In this case, then, the intermediate, $\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}$, and the end product, $\left(\mathrm{HbO}_{2}\right)_{2}$, are absent at $\mathrm{t}=0$. The progress curve monitored at an appropriate wavelength in the $\mathrm{O}_{2}$ difference spectrum. Such monitoring records the time dependence of all intermediates, simultaneously. The time course obtained by spectroscopic procedures, then, is directly proportional to fractional saturation of $\mathrm{Hb}_{2}$ with $\mathrm{O}_{2}$. The equation of state for fractional saturation of

[^0]$\mathrm{Hb}_{2}$ binding sites with $\mathrm{O}_{2}$ is
$F=\frac{\left[\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}\right]+2\left[\left(\mathrm{HbO}_{2}\right)_{2}\right]}{2\left[\mathrm{Hb}_{2}\right]_{0}}$
A rate law based on fractional saturation is readily obtained by substituting the individual rate laws for $\left(\mathrm{HbO}_{2}\right) \mathrm{Hb}$ and $\left(\mathrm{HbO}_{2}\right)_{2}$ presented above.
$\mathrm{F}=\frac{\left[\mathrm{Hb}_{2}\right]_{0}\left(2 \exp \left(-k_{2} t\right)-2 \exp \left(-k_{1} t\right)\right)+2\left[\mathrm{Hb}_{2}\right]_{0}\left(1+\exp \left(-k_{1} t\right)-2 \exp \left(-k_{2} t\right)\right)}{2\left[\mathrm{Hb}_{2}\right]_{0}}$
Factoring out common terms and combining similar quantities, the rate law reduces to a first order equation.
$\mathrm{F}=1-\exp \left(-k_{2} t\right)$


[^0]:    Correspondence to: Francis Knowles, Department of Chemistry and Biochemistry, University of California, San Diego, La Jolla, USA, E-mail: fknowles@ucsd.edu
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