APPENDIX

Derivation of a mathematical model for propagation of a horizontal fracture in a frac-packed gas hydrate reservoir

This section provides a derivation of an analytical model for predicting the propagation of a horizontal fracture in frac-packed gas well in a gas hydrate reservoir.

Assumptions

The following assumptions were made in model formulation.

- The fracturing fluid carrying sand/proppant is a Newtonian fluid. This assumption is valid in frac-packing operations with water as fracturing fluids.
- The fracture takes a circular shape with a constant width. This assumption is valid because the fracture width is much less than the fracture radius.
- The fluid leak-off during frac-packing is negligible. This assumption is valid because the pore space in the hydrate zone is filled by solid hydrates during the frac-packing.

Governing equation

Consider radial flow through a volume element in a horizontal fracture depicted in Figure 1.

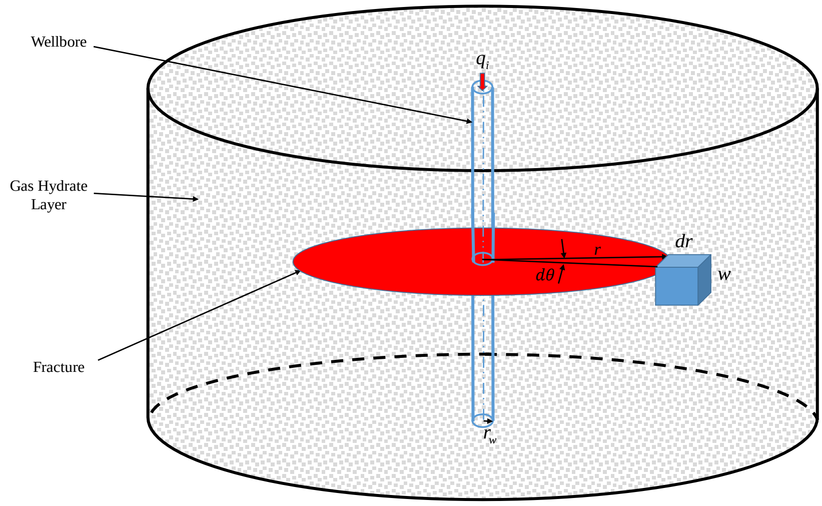

Figure 1: Propagation of a horizontal fracture

Hydraulics gives the following relation:

$$dp = -\frac{2 f_F \rho_L}{g w} v^2 dr \qquad (A.1)$$

where p is the pressure in lbf/ft2; fF is the Fanning friction factor; ρL is the fluid density in lbm/ft3; g is the gravitational acceleration factor (32.17 lbm-ft/lbf-s2); ν is the velocity of fluid (or the fracture propagation rate) in ft/s; r is the fracture propagation radius in ft; and w is the average fracture width in ft [26]. The average fracture width may be estimated by the equation presented by Geertsma and Klerk (1969) for radial fractures.

If the fracturing fluid is injected into only one fracture, the injection rate q_i is expressed as

$$q_i = \int_0^{2\pi} vwrd\theta = 2\pi wrv \qquad \text{(A.2)}$$

which gives

$$v = \frac{q_i}{2\pi wr} \qquad \text{(A.3)}$$

Substituting Eq. (A.3) into Eq. (A.1) gives

$$dp = -\frac{f_F \rho_L q_i^2}{2\pi^2 g w^3 r^2} dr \qquad \text{(A.4)}$$

which is integrated to give

$$\int_{p_F}^{p_t} dp = \int_{r_w}^{r} \frac{-f_F \rho_L q_i^2}{2\pi^2 g w^3 r^2} dr \qquad \text{(A.5)}$$

or

$$p_F = p_t + \frac{f_F \rho_L q_i^2}{2\pi^2 g w^3} \left(\frac{1}{r_w} - \frac{1}{r}\right) \qquad \text{(A.6)}$$

Unit conversion: The above-derived equations are only valid for U.S. engineering units. U.S. oilfield units are used in hydraulic fracturing design where pressure is in psi, the flow rate is in bpm, fracture width is in inch, and distance is in feet. Equation (A.6) takes the form in US oilfield units

$$p_F = p_t + \frac{1.66 \times 10^{-4} f_F \rho_L q_i^2}{w^3} \left(\frac{1}{r_w} - \frac{1}{r}\right) \qquad \text{(A.7)}$$