

Optimization of Autofrettage-Reautofrettage Percent and Shrink-Fit Combination for Optimum Fatigue Life in Multilayer Pressure Vessels

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Abstract

With the increase in demand of materials for production, it is required to design an object such that consumption of material can be reduced for same working conditions or load carrying capacity can be enhanced for same consumption of material. In this paper, pressure vessel has been designed to withstand high internal pressure for same consumption of material. For this purpose, multilayer pressure vessel is constructed by shrink-fit of vessel, which has residual stress within it caused by pre-autofrettage, followed by autofrettage of the assembly. Thus, increase in the number of layers increases the number of ways to assemble and hence this analysis is limited to three-layered pressure vessel. Possible ways to assemble the three-layered vessel for different autofrettage or re-autofrettage percent and radial interference for shrink-fit are discussed in this paper. Considering these parameters as design parameters and maximization of fatigue life as an objective function, optimization is carried out by Genetic Algorithm. For the optimum value of design variables, stress distribution for the assembled pressure vessel at design pressure is studied. It is observed that fatigue life of each case is different with little variation of maximum and minimum hoop stresses.

Keywords: Autofrettage; Fatigue life; Genetic Algorithm (GA); Multilayer vessel; Re-autofrettage; Shrink-fit

Introduction

For effective application of multilayer pressure vessel, different layers of optimum thickness require to be shrink-fitted to optimum degree of interference [1,2]. Effective application of pressure vessel means increasing the fatigue life and decreasing the resulting stresses throughout the radial positions for the same consumption of material. But fatigue life can be increased by decreasing resulting stress (i.e. effective hoop stress) at the innermost surface, for which, residual hoop stress needs to be highly compressive (almost equal to working hoop stress). But it is observed that due to autofretting or shrink-fitting, residual stresses generated at the inner and outer surface of the vessel (outer layer in case of shrink-fitting) are of compressive and tensile in nature respectively. While optimizing the fatigue life, it is observed that sometimes (for some values of autofrettage percent and shrink-fit) net resulting hoop stress at the outer or middle layer is subjected to extreme tensile stress, which exceeds the design limit. Therefore, the resulting hoop stress distribution during optimization needs to be constrained by maximum value of design hoop stress. Design pressure for multilayered vessel is calculated from ASME codes (KD-221) [3].

This process is applied to various options of assembly. Some of them are studied by Jahed et al. [4]. Based on number of times autofrettage is required for assembly, various ways to assemble the three-layered vessel is possible. Some of the possible effective options are:

a) Perform autofrettage on each layer (inner, middle, outer) individually. Then shrink-fit second layer on first layer followed by shrink-fit of third layer on first sub-assembly.

b) Perform autofrettage on first two layers (inner and middle layer). Then shrink-fit second layer on first layer followed by shrink-fit of third layer on first sub-assembly.

c) Perform autofrettage on each layer (inner, middle, outer) individually. Then shrink-fit second layer on first layer followed by autofrettage (re-autofrettage) of sub-assembly. Finally, shrink-fit third layer on the sub-assembly followed by autofrettage of final combination.

d) Perform autofrettage on the inner and outer layers individually and then shrink-fit all layers followed by final autofrettage of the assembly.

e) Perform autofrettage on the inner and outer layers individually. Shrink fit middle layer (non-autofretted) on the inner layer (autofretted) followed by autofrettage of sub-assembly. Finally, shrink-fit third autofretted layer on the sub-assembly.

f) Perform autofrettage on each layer individually and then shrink-fit all layers sequentially. Finally, autofrettage the assembly obtained.

g) Perform autofrettage in each layers individually followed by shrink fit of first two layers. Then autofrettage the sub-assembly followed by shrink-fit of third layer on the sub-assembly.

h) Shrink-fit first two layers followed by autofrettage of the sub-assembly. Then shrink-fit the third autofretted layer on the sub-assembly.

i) Shrink-fitting all the non-autofretted layers followed by final autofrettage of the assembly.

j) Shrink-fitting the third non-autofretted layer on the first two non-autofretted shrink-fitted combination.

In the study on Autofrettage, Huang and Moan [5] calculated residual stress of monobloc vessel considering Bauschinger Effect. Bauschinger effect is the condition when the yield strength of a metal

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Received August 22, 2012; **Accepted** September 24, 2012; **Published** September 26, 2012

Citation: Kumar N (2012) Optimization of Autofrettage-Reautofrettage Percent and Shrink-Fit Combination for Optimum Fatigue Life in Multilayer Pressure Vessels. J Appl Mech Eng 1:113. doi:10.4172/2168-9873.1000113

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decreases as a result of extreme loading pressure. The behavior of stress-strain curve becomes non-linear. Thus the loading of pressure vessel is linear while the unloading becomes non-linear, if Bauschinger effect is taken into consideration.

But as far as multilayer pressure vessel is concerned, Bauschinger Effect is not much effective. It is due to the fact that thickness and ratio of outer radius and inner radius of each layer becomes less than two [6] and hence critical pressure required to generate plastic zone [7] is much higher than maximum limit of autofrettage pressure as computed by Majzoobi and Ghomi [8] and Blazinski [9]. Residual stress calculation of monobloc vessel (neglecting Bauschinger effect) is studied by Ayob and Elbasheer [10] and Hill [6].

Radial deformation developed due to shrink-fit and autofrettage of each layer is neglected. It is assumed that the strain produced on the outer surface of combination is regained to its original dimensions by surface finish operation.

Re-autofrettage in this case is limited to autofrettage of sub-assembly and final assembly obtained after shrink-fitting (e.g. case (b) to case (g)). Net effect of autofrettage and re-autofrettage is calculated by summation of the residual stress distribution along thickness, generated by autofrettage or re-autofrettage, because stress generated due to autofrettage and re-autofrettage is much below yield point. Hence, effective hoop stress for maximum fatigue life is the combined effect of autofrettage, re-autofrettage and shrink-fit. All analysis including autofrettage, re-autofrettage and shrink-fit of single layer and optimization is done with help of *Matlab-7*.

Introduction to genetic algorithm

Genetic Algorithm is a class of probabilistic optimization algorithm, which is based on steps of natural genetics and introduced by Professor John Holland of University of Michigan in 1975 [11]. Random search by GA is suitable for nonlinear problems.

Genetic algorithm is computerized search algorithm. It performs a random search and is suitable for nonlinear problems. For GA solution, first of all an objective function (f) is defined and then depending on minimization or maximization problem, fitness function is defined. If the objective function is to be maximized, then fitness function $F(x)=f$, while for minimization the objective function $F(x)=1/(1+f)$. Then, design variables (x_i) are selected, value of which will vary for the optimum value of fitness function.

Basic steps are:

- The operation of GAs begins with a population of random strings representing design variables (x_i).
- A random search of a defined N-dimensional solution space is performed to obtain the string, which consists of values of design variables which are within the given range as specified by user.
- The fitness value for each string is evaluated.
- The best fitness value of the population is selected for *reproduction* to form the next generation.
- The population is then operated by three main operators *reproduction, crossover, and mutation*, to create new population of points. The new population is related to the old one in a particular way. The new population is further evaluated and tested for termination. If the termination criterion is not met, the population is iteratively operated by the above three operators and evaluated. The iteration

process is continued until pre-specified number of generations is complete.

For formulation of GA algorithm, thickness of each layer, autofrettage percentage of each layer, radial interference for shrink-fit and autofrettage or re-autofrettage percentage of the sub-assembly or assembly are considered as design variables, whereas maximization of fatigue life is considered as objective function. The design variables are varied within feasible limit for maximizing fatigue life. To obtain optimum thickness, mating surfaces are varied from inner radius to outermost radius such that radial position of second mating surface is greater than first. Autofrettage or re-autofrettage percent is varied from 0 to 100 %, whereas interference-fit is varied from 0 to 0.5 mm. Only those combinations of design variables which generate maximum working hoop stress throughout the thickness less than design stress limit is taken into consideration for fatigue life evaluation. Working pressure is assumed equal to design pressure as calculated from ASME codes (KD-221).

Mathematical Model

Working stress distribution is computed by Lamé's equation, which is expressed as:

$$\begin{aligned}\sigma_r &= p * \frac{r_i^2 / r_o^2}{1 - r_i^2 / r_o^2} * \left(1 - \frac{r_o^2}{r^2}\right) \\ \sigma_t &= p * \frac{r_i^2 / r_o^2}{1 - r_i^2 / r_o^2} * \left(1 + \frac{r_o^2}{r^2}\right) \\ \sigma_z &= p * \frac{r_i^2 / r_o^2}{1 - r_i^2 / r_o^2}\end{aligned}\quad (1)$$

Residual stress analysis due to autofrettage- reautofrettage

Due to autofrettage of monobloc vessel, plastic and elastic regions are generated. Inner part is stressed beyond elastic limit while the remaining outer part is within elastic region. Stress due to autofrettage is computed by using following equations.

In plastic region ($r_i \leq r \leq r_c$):

$$\begin{aligned}\sigma_{pr} &= \frac{Y}{2} \left[\left\{ 2 \log \left(\frac{r}{r_c} \right) - 1 + \frac{r_c^2}{r_o^2} \right\} - \left\{ 2 \log \left(\frac{r_c}{r_o} \right) + 1 - \frac{r_c^2}{r_o^2} \right\} \right] \left(\frac{1}{K^2 - 1} \right) \left(1 - \frac{r_o^2}{r^2} \right) \\ \sigma_{pr} &= \frac{Y}{2} \left[\left\{ 2 + 2 \log \left(\frac{r}{r_c} \right) - 1 + \frac{r_c^2}{r_o^2} \right\} - \left\{ 2 \log \left(\frac{r_c}{r_o} \right) + 1 - \frac{r_c^2}{r_o^2} \right\} \right] \left(\frac{1}{K^2 - 1} \right) \left(1 + \frac{r_o^2}{r^2} \right) \\ \sigma_{pz} &= \frac{Y}{2} \left[\left\{ 1 + 2 \log \left(\frac{r}{r_c} \right) - 1 + \frac{r_c^2}{r_o^2} \right\} - \left\{ 2 \log \left(\frac{r_c}{r_o} \right) + 1 - \frac{r_c^2}{r_o^2} \right\} \right] \left(\frac{1}{K^2 - 1} \right)\end{aligned}\quad (2)$$

In elastic region ($r_c \leq r \leq r_o$):

$$\begin{aligned}\sigma_{er} &= \frac{Y}{2} \left(1 - \frac{r_o^2}{r^2} \right) \left[\frac{r_c^2}{r_o^2} - \left\{ 1 - \frac{r_c^2}{r_o^2} + 2 \log \left(\frac{r_c}{r_o} \right) \right\} \right] \left(\frac{1}{K^2 - 1} \right) \\ \sigma_{er} &= \frac{Y}{2} \left(1 + \frac{r_o^2}{r^2} \right) \left[\frac{r_c^2}{r_o^2} - \left\{ 1 - \frac{r_c^2}{r_o^2} + 2 \log \left(\frac{r_c}{r_o} \right) \right\} \right] \left(\frac{1}{K^2 - 1} \right) \\ \sigma_{er} &= \frac{Y}{2} \left[\frac{r_c^2}{r_o^2} - \left\{ 1 - \frac{r_c^2}{r_o^2} + 2 \log \left(\frac{r_c}{r_o} \right) \right\} \right] \left(\frac{1}{K^2 - 1} \right)\end{aligned}\quad (3)$$

Residual stress analysis due to shrink-fit

Residual stress can also be generated by introducing interference fit. Stress calculation due to interference fit is studied by Harvey [12]. Fitting pressure generation due to radial interference (δ) is expressed as:

$$P_{int} = \frac{\delta}{\frac{r_{1i}}{E_1} \left(\frac{r_{1o}^2 + r_{1i}^2}{r_{1o}^2 - r_{1i}^2} - \nu_1 \right) + \frac{r_{2i}}{E_2} \left(\frac{r_{2o}^2 + r_{2i}^2}{r_{2o}^2 - r_{2i}^2} - \nu_2 \right)} \quad (4)$$

Residual stress distribution due to shrink-fit is obtained by applying the fitting pressure to Lamé's equation for each layer separately.

Fatigue life calculation

Fatigue life is the number of cycles causing the initial crack to propagate upto its critical dimensions. Calculation of critical crack length is based on ASME codes (KD-421). Loading is of repeated in nature i.e. working pressure is varied from zero to maximum and maximum to zero. The fatigue crack growth of a material subjected to variable loading is studied by Paris [13]. Paris law is accredited as:

$$\frac{da}{dN} = C * \Delta K^n \quad (5)$$

ΔK is the range of stress intensity factor which is given by,

$$\Delta K = \Delta \sigma * (\pi a)^{1/2} \quad (5a)$$

$\Delta \sigma$ of equation (5a) is the positive range of the sum of Lamé hoop

stress (depending on nature of cyclic loading) and residual hoop stress at the innermost surface.

$$\text{i.e. } \Delta \sigma = \Delta \sigma_{Lam\acute{e}(Hoop)} + \Delta \sigma_{Residual, Hoop} \quad (6)$$

Considering equation (5) & (5a), Paris equation takes the form:

$$N = \frac{1}{C * \pi^{n/2} * (1-n/2) * \Delta \sigma^n} [a_f^{(1-n/2)} - a_o^{(1-n/2)}] \quad (7)$$

Where, α_o and α_f are calculated from ASME code standards (KD-421). From equation (7) number of fatigue cycles can be calculated.

Results and Discussion

The analysis is performed on a three-layered cylinder. Geometry and mechanical properties of layered vessel are:

- Innermost radius: $r_i = 90$ mm,

- Outermost radius: $r_o = 140$ mm,

- Working pressure: $P = 332.847$ 3 MPa.

- Multilayer vessel whose material is A723 steel with the following mechanical properties:

- $E = 209$ GPa,

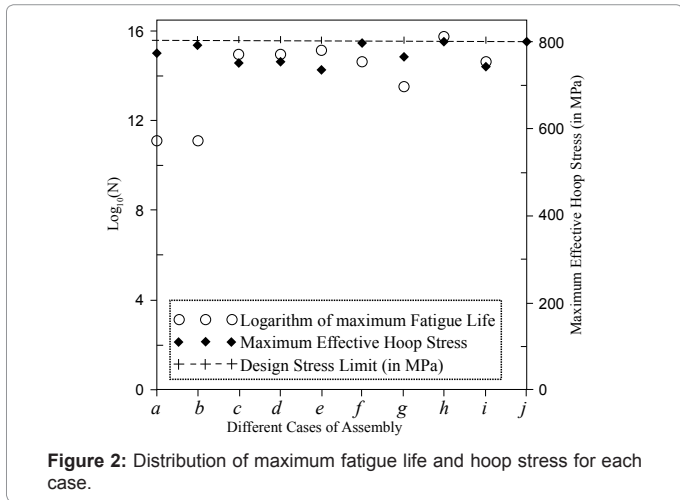
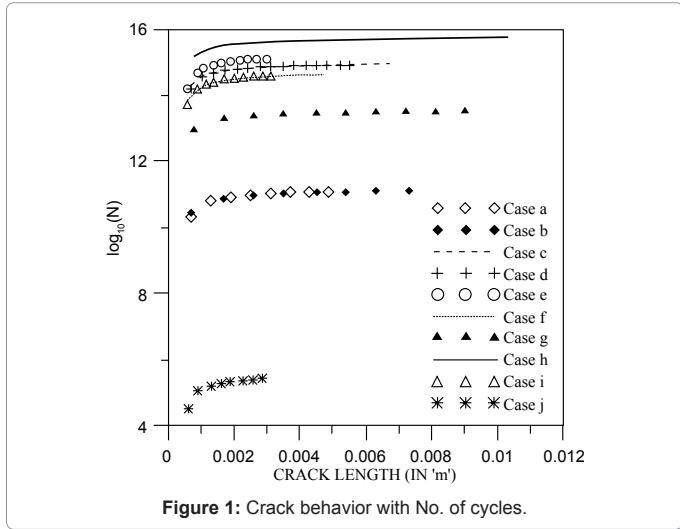
- $\nu = 0.3$,

- $Y = 1$ 130 MPa.

- Fracture parameters of A723 are:

CASES	THICKNESS OF INDIVIDUAL LAYER (in mm)	AUTOFRETTAGE IN INDIVIDUAL LAYER		AUTOFRETTAGE IN SUB-ASSEMBLED		RADIAL INTERFERENCE (mm) AND SHRINK-FIT PRESSURE (MPa)		Critical crack-length (in m)	FATIGUE LIFE
		Percent	Pressure	Percent	Pressure	Radial Interference	Contact pressure		
a	5.0003	16.13	58.8640	0	0	0.3871	39.7397	0.005	1.2366E+11
	39.9994	45.16	365.3991	0	0	0.0484	2.5115		
	5.0003	87.10	41.0740						
b	7.5005	16.13	85.6435	0	0	0.4032	54.6229	0.0075	1.3086E+11
	34.9990	35.48	311.6386	0	0	0.1290	9.8995		
	7.5005	0	0						
c	6.9448	61.29	83.1181	35.48	388.4566	0.1129	20.9331	0.0069	9.17E+14
	36.1104	9.68	283.0639	16.13	384.7602	0.1290	5.9354		
	6.9448	96.77	57.4897						
d	5.7146	22.58	67.1111	0	0	0.1774	20.0645	0.0057	8.95E+14
	38.5708	0	0	45.16	452.6042	0.0645	3.8102		
	5.7146	41.94	46.4528						
e	3.1252	35.48	38.0389	45.16	430.9974	0.1774	12.4383	0.0031	1.33E+15
	43.7496	0	0	0	0	0.1129	3.8115		
	3.1252	22.58	25.1710						
f	18.7501	96.77	213.8094	0	0	0.1129	14.9865	0.0047	4.21E+14
	12.4998	58.06	120.8019	35.48	433.6926	0.1774	29.4731		
	18.7501	29.03	151.9245						
g	9.6158	87.10	114.5320	25.81	353.6505	0.2419	37.1952	0.0096	3.32E+13
	30.7684	41.94	281.8590	0	0	0.1613	15.5374		
	9.6158	54.84	79.3089						
h	10.2563	0	0	45.16	379.3672	0.2419	38.1494	0.0103	5.9E+15
	29.4874	0	0	0	0	0.1936	19.7933		
	10.2563	25.81	82.5703						
i	3.3335	0	0	0	0	0.1774	13.1389	0.0033	4.06E+14
	43.3330	0	0	45.16	452.6042	0.1129	4.0504		
	3.3335	0	0						
j	3.0305	0	0	0	0	0.3065	20.9331	0.003	2.849E+05
	43.9390	0	0	0	0	0.1774	5.9354		
	3.0305	0	0						

Table 1: Critical crack-length and Maximum Fatigue Life for optimum design variables of all cases.



$$C = 2.41e-11 \text{MPa}^{-1} \text{m}^{1/2}$$

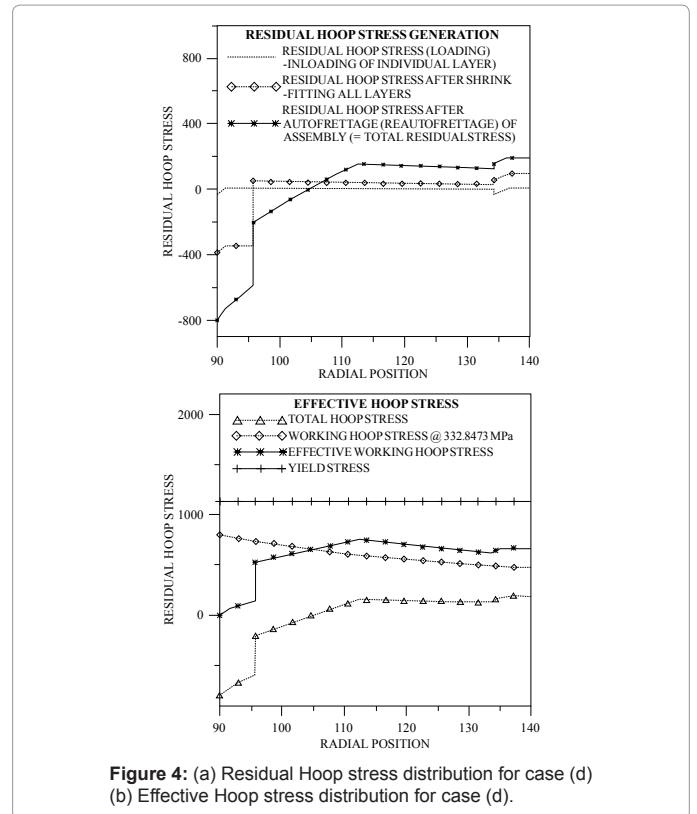
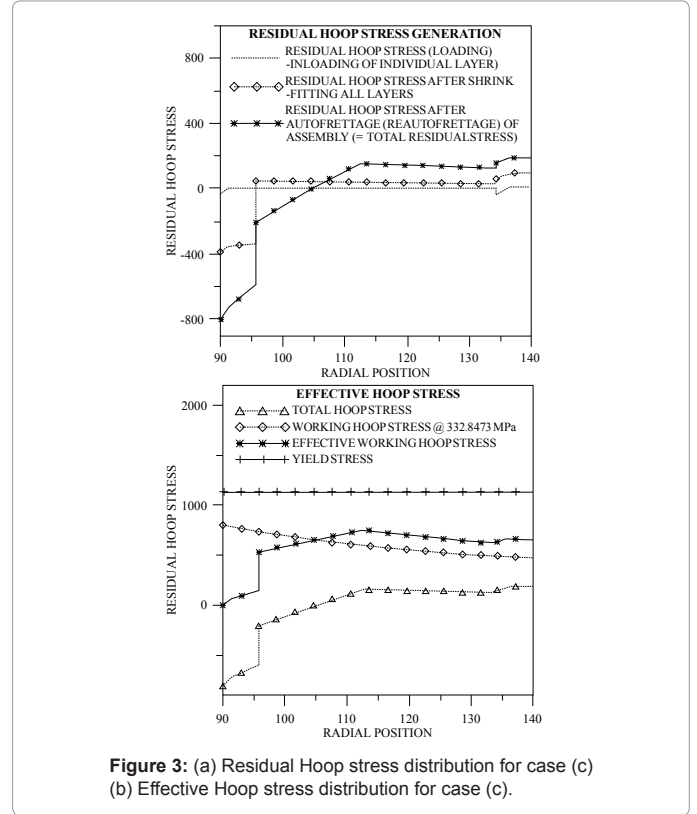
$$n = 2.80$$

In the first step, the assembly parameters are optimized for maximum value of fatigue life for each options of assembly, which are discussed previously. Then the maximum fatigue life for each case compared to find the best way to assemble.

Optimization of fatigue life

First of all, Genetic Algorithm is used to maximize the fatigue life for a specific set of assembly parameters, which are obtained as optimum value of design variables in GA. For different cases, number of design variables varies, which depend on number of vessels (or layers) and number of autofrettage process in each vessel or sub-assembly or assembly of vessels. Number of design variables for thickness and radial interference for three-layered vessel is constant. Therefore total number of design variables depends on number of times autofrettage are required for assembly. e.g. number of design variables for thickness and shrink-fit in three-layered pressure vessel are three & two respectively. Therefore, number of design variables for autofrettage in case (a) are three and thus total number of design variables are eight (three for thickness, three for autofrettage of each layer & two for radial

interference for shrink-fit). Similarly, number of design variables for autofrettage in case (c) are five (first three for autofrettage of each layer,



fourth is for autofrettage of the shrink-fitted subassembly of two layers and fifth one is for autofrettage of the assembly of three layers) and

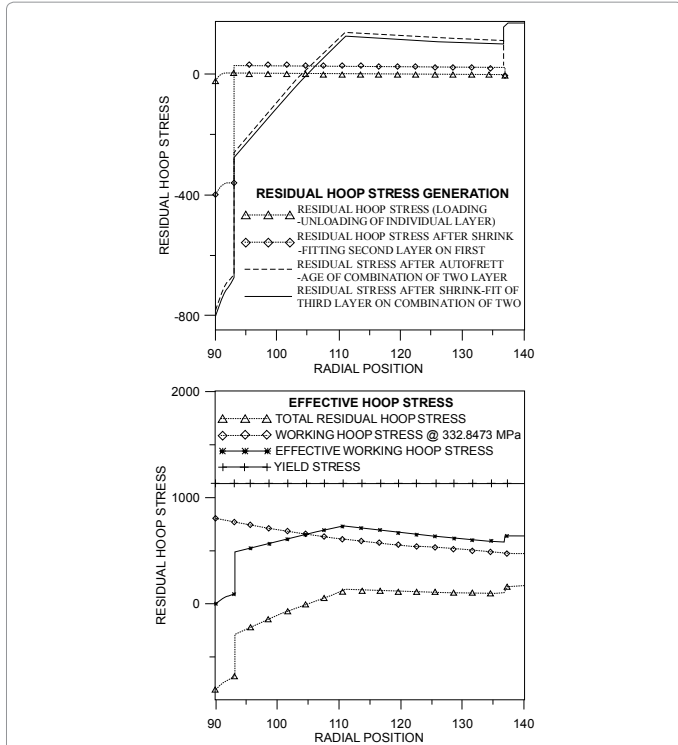


Figure 5: (a) Residual Hoop stress distribution for case (e) (b) Effective Hoop stress distribution for case (e).

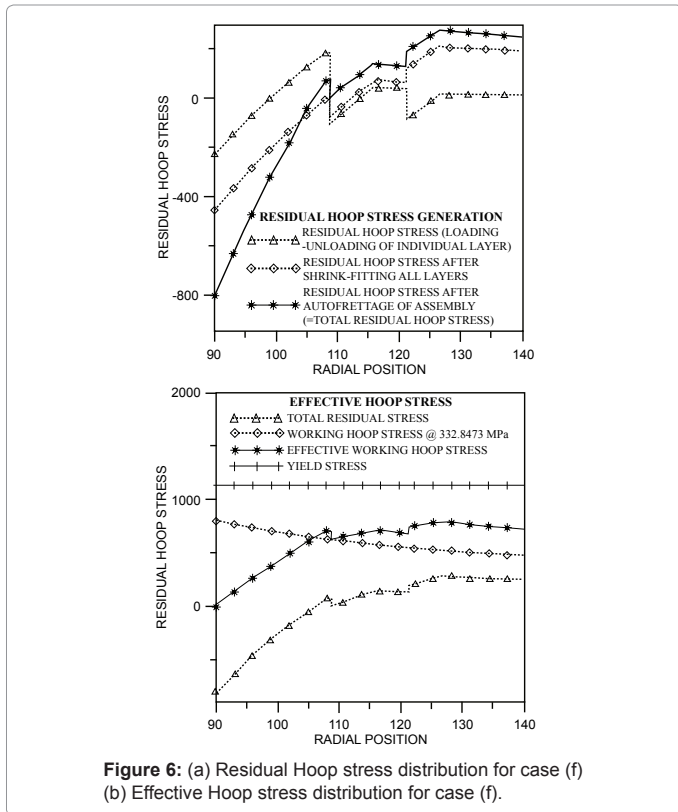


Figure 6: (a) Residual Hoop stress distribution for case (f) (b) Effective Hoop stress distribution for case (f).

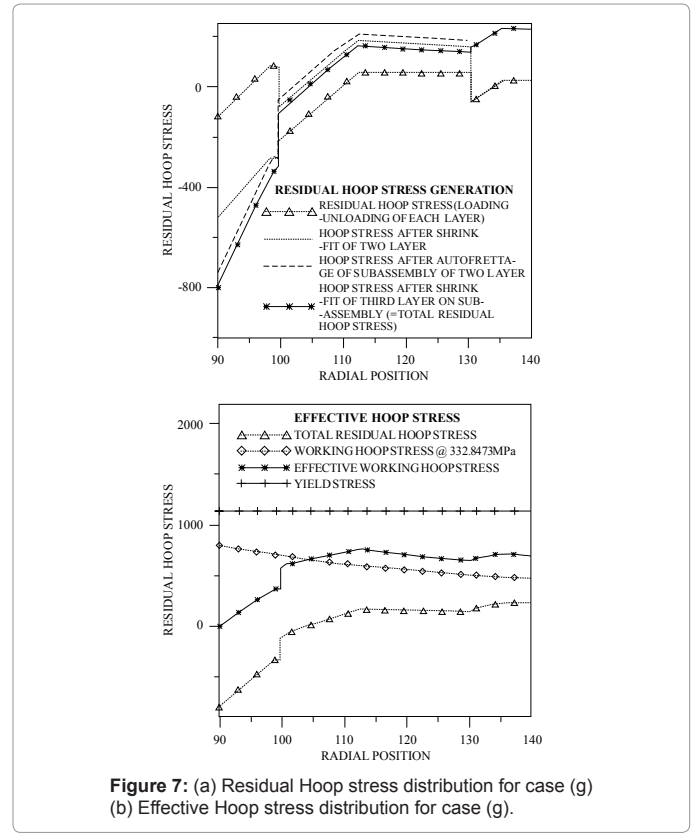


Figure 7: (a) Residual Hoop stress distribution for case (g) (b) Effective Hoop stress distribution for case (g).

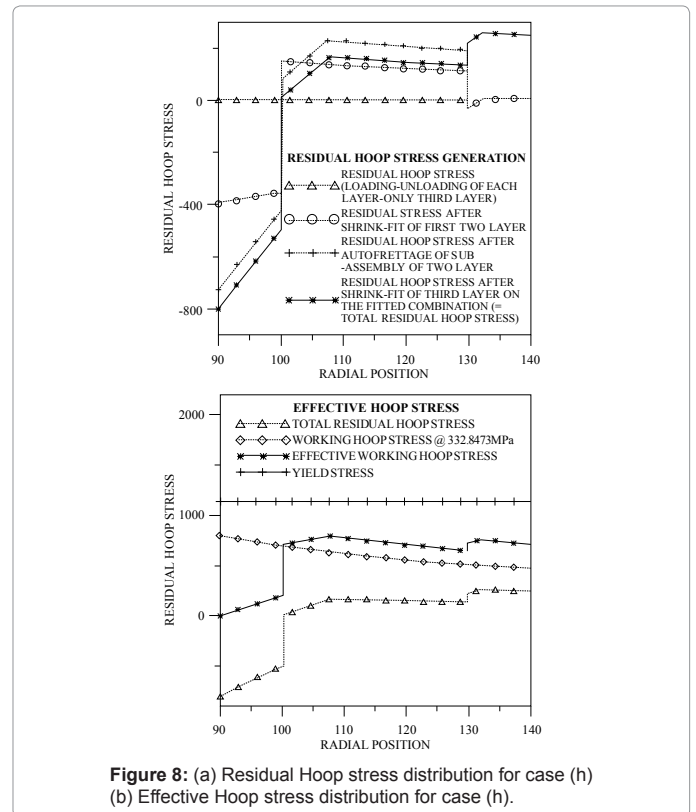


Figure 8: (a) Residual Hoop stress distribution for case (h) (b) Effective Hoop stress distribution for case (h).

thus total number of design variables are ten. In similar way, number of required assembly parameters is calculated for each case.

In each iteration of GA, design variables are varied and corresponding fatigue life is calculated. Calculation of fatigue life is discussed in "Fatigue life calculation". The set of design variables which

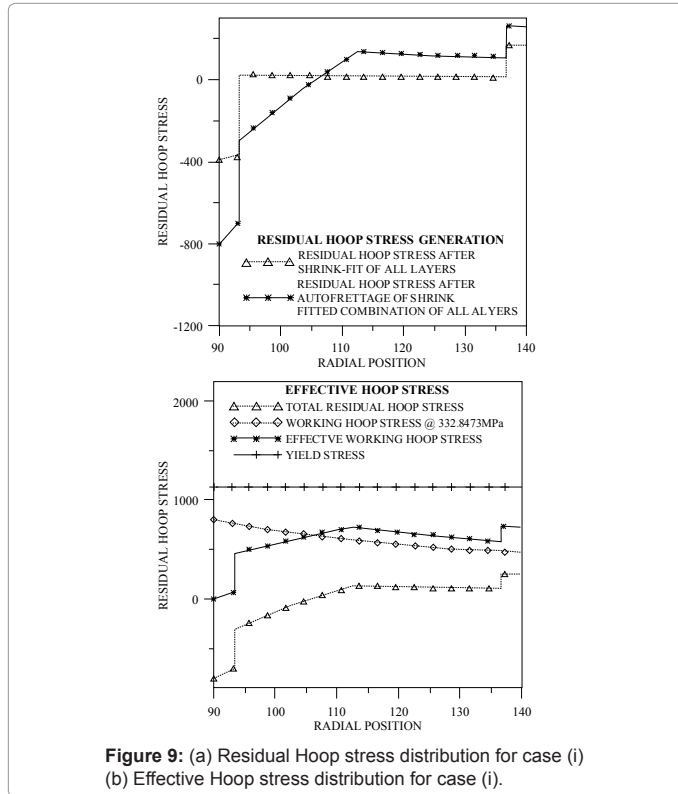


Figure 9: (a) Residual Hoop stress distribution for case (i) (b) Effective Hoop stress distribution for case (i).

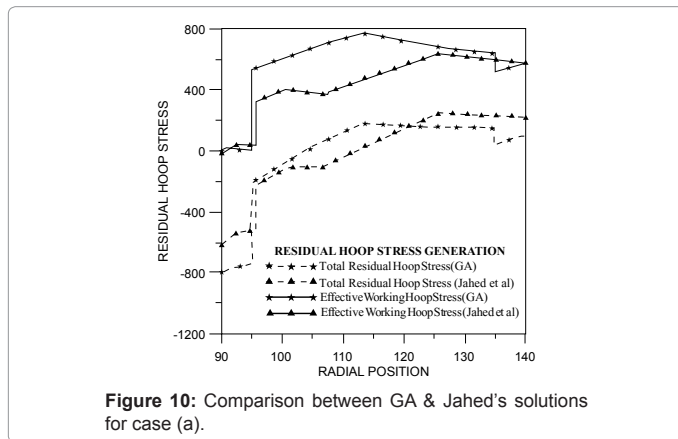


Figure 10: Comparison between GA & Jahed's solutions for case (a).

gives maximum fatigue life in each generation is stored for comparison in next generation. In this way, best solution is obtained. The design variables of the best solution are known as optimum solution. Optimum parameters for each case and their respective fatigue life are tabulated in (Table 1).

Optimum thickness, autofrettage percent of each layer, re-autofrettage or autofrettage percent of subassembly or assembly and radial interference for shrink-fit is presented in (Table 1). (Table 1) also shows the autofrettage, re-autofrettage and contact pressure (in MPa) for the corresponding autofrettage-reautofrettage percent and radial interference respectively. Apart from this, respective critical crack length and fatigue life is shown in (Table 1) for each case.

From (Table 1), it is observed that for some of the cases, fatigue life is nearly same with some exceptions. Hence effective case is sorted out depending on types of assembly and fatigue life obtained. Types of assembly mean number of times autofrettage is required for assembly in a particular case. e.g. number of autofrettage process required for assembly in case (a) is three whereas that in case (c) is five. Thus, effectiveness is based on minimum number of autofrettage process and maximum fatigue life. Based on this idea, "case (h)" is the most effective option for assembly of multilayer pressure vessel.

Nature of propagation of crack with number of cycles is plotted in (Figure 1) for all cases. (Figure 1) illustrates variation of crack length with logarithmic value of number of fatigue cycles for each case.

For different loading conditions, different thickness, autofrettage percentage and shrink-fit are required for maximum fatigue life. It is due the reason that for a specific loading condition, assembly parameters required to generate compressive residual hoop stress at inner surface equal to the working hoop stress at the same surface will be different.

Stress distribution for each case

Hoop stress distribution for the optimum set of design variables is plotted for the effective cases. Effectiveness is sorted out from (Figure 2), which illustrates the maximum fatigue life ($\log_{10}(N)$) and effective hoop stress for each case. All cases are numbered sequentially and are represented on x-axis. From (Figure 2, case (j)) is not effective in increasing the fatigue life though it reduces effective hoop stress. Thus, if fatigue life is not a dominating factor i.e. the vessel is under static loading or amplitude of fluctuation of load is less, then case (j) is useful, otherwise this option for assembling the layered-pressure vessel is discarded. But if both fatigue life and effective hoop stress are design factors, means if both need to care while designing, then case (e) is beneficial. Similarly, case (h) is effective for obtaining the maximum fatigue life. However, resulting hoop stress in case (h) is comparatively greater than that in case (e). If the material can permit that much increase in hoop stress then case (h) is the best option. Otherwise case (e) is to be opted for design of three-layered pressure vessel. The two possible effective ways to design multilayer pressure vessel are case (e) & case (h), which are to be selected by the designer. Hence, design of multilayer vessel is based on working conditions and the material properties.

Stress distribution is based on equation (1) to equation (4). For optimum assembly parameters, residual stress generation is based on equation (2), (3), & (4). Effective stress is plotted by summation of working stress (obtained from equation (1)) and residual stress distribution throughout the thickness. (Figures 3-9) show residual and effective working hoop stress generation for some of the effective cases (case (c) to (i)). Case (f) is the only method of assembly, which produces almost continuous nature of residual and effective stress curve. Referring to (Figure 6(a)), nature of residual hoop stress curve is nearer to mirror image of working stress. Hence, this method of assembly may be beneficial for extremely high working pressure.

Maximum and minimum effective hoop stress for each case is shown in (Table 2). Maximum hoop stress for each case is constrained by the maximum hoop stress in monobloc vessel of same equivalent thickness, due to working pressure equal to design pressure. From this table, it is clear that maximum and minimum hoop stresses for all cases are closer except case (j), in which the layered-pressure vessel is assembled by shrink-fit only. Thus design of multilayer vessel becomes more effective if combine effect of autofrettage and shrink-fit is used.

	Case a	Case b	Case c	Case d	Case e	Case f	Case g	Case h	Case i	Case j
MINIMUM EFFECTIVE HOOP STRESS	-1.3093	-1.3271	-0.0558	0.0554	-0.0453	-0.0713	-0.1869	-0.0296	0.0695	127.5867
MAXIMUM EFFECTIVE HOOP STRESS	772.3309	792.4510	750.6875	754.3461	733.7209	795.7023	761.5589	798.0989	743.4437	800.0429

Table 2: Maximum and minimum hoop stress (in MPa) at design pressure for all cases for optimum thickness, autofrettage percent and shrink-fit.

Validation

Jahed's analysis is plotted in (Figure 10) and compared with the stress distribution for the set of parameters obtained from GA. Jahed's plot for case (a) is obtained by putting the optimum parameters of simplex method in the *Matlab-program*, which is same as that in Jahed's paper. This validates the basic *Matlab-programming*.

Conclusion

In present work, optimization of different parameters by Genetic Algorithm is carried out and following conclusions are derived:

I) For three layered vessel, optimum condition of fatigue life or stress is achieved if thickness of first and third layer are equal and thickness of middle layer is greater than other layers.

II) If fatigue life is given importance i.e. working pressure for the vessel to be designed is variable then assembly of three-layered pressure vessel needs to be done according to case (h). But maximum effective hoop stress for this case is comparatively.

III) For the vessel under high working pressure, case (e) is beneficial

in respect to fatigue life which is nearer to that of case (h) as effective stress in case (e) is less than case (h).

IV) Complication to assemble the three-layer pressure vessel for Case (i) is less. Also fatigue life is up to the standard and lie in between the best two solutions (case (h) & case (e)).

V) Case (e) & case (h) are the best options to assemble three-layer pressure vessel. But selection of process to assemble layered vessel depends on the requirement of designer.

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Nomenclature

A	: Crack length.
α_f	: Critical crack length.
α_o	: Initial crack length.
C, n	: Material properties known as fracture parameter.
rc	: Plastic radius during loading.
$E1, E2$: Young's modulus of 1 st and 2 nd layer respectively.
K	: Radial ratio= outer radius/Inner radius.
N	: Number of cycles.
P_{int}	: Contact pressure due to interference fit.
r_i	: Inner radius of pressure vessel.
r_o	: Outer radius of pressure vessel.
r_{1i}	: Inner radius of 1 st layer.
r_{1o}	: Outer radius of 1 st layer.
r_{2i}	: Inner radius of 2 nd layer.
r_{2o}	: Outer radius of 2 nd layer.
Y	: Yield stress.
$\Delta\sigma$: Effective hoop stress at the innermost surface under cyclic loading condition, which reflects positive range of the sum of Lamé hoop stress and residual hoop stress at innermost surface.
$\Delta\sigma_{Lame(Hoop)}$: Range of working hoop stress depending on no. of cycles.
$\sigma_{Residual, Hoop}$: Residual hoop stress at innermost surface.
δ	: Radial interference ($=r_{1o} - r_{2i}$).
σ_{erR}	: Residual radial stress in elastic zone.
σ_{elR}	: Residual hoop stress in elastic zone.
σ_{erR}	: Residual radial stress in elastic zone.
σ_{prR}	: Residual radial stress in plastic zone.
σ_{ptR}	: Residual hoop stress in plastic zone.
σ_{pzR}	: Residual axial stress in plastic zone.
V_1, V_2	: Poisson's ratio of 1 st & 2 nd vessel respectively.