Instantaneous Power and Phase Shifts at Piezoceramic Bars Forced Vibrations

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Abstract

This paper is devoted to analysis of the electric loading conditions problem for piezoceramic bars' forced vibrations. New simple experimental technique together with computing permits to investigate many resonators' parameters: admittance, impedance, phase shifts, power components etc for constant amplitude input voltage, constant amplitude sample voltage and constant amplitude sample current electric conditions based on experimental data for "as there is" regime. Such computer modeling makes possible to decrease of the experimental difficulties and to study in linear approximations the dependence of resonators' parameters from loading conditions. The fundamental mode of vibrations of thin piezoelectric bar is given as example. It is established that high admittance nonlinearity in constant amplitude voltage regime and its absence for constant amplitude current case are created by different behavior of instantaneous power level. The obtained results are in good agreement with experimental data.

Introduction

Piezoceramic constructive elements' vibrations characterize by great electromechanical coupling between electric fields and elastic displacements or stresses [1-5]. The internal physical processes nature in such bodies drives to the fact that mechanical displacements, strains and stresses, sample admittance, impedance or instantaneous power have both real and imaginary parts. Thus it is possible to calculate any amplitude with accounting the energy losses only. The analytic solutions for electro-elastic vibrations of simple geometric form bodies are used for determination the real parts of the dielectric, elastic and piezoelectric coefficients [6-8]. Imaginary parts usually are determined on maxima/minima admittance, first proposed by Martin [9]. Very important role in energy losses belongs to mechanic quality factor Q, which differs for resonance and anti-resonance [10].

Acoustic bulletin [11] was devoted to modeling of the loss-energy piezoceramic resonators by electric equivalent networks Van-Dyke-type with passive elements. It was shown that when piezoelectric sample is excited by constant voltage the instantaneous power in sample increases at resonance frequency in many times in respect to off-resonant case. And when sample is excited by constant current the instantaneous power in sample decreases at resonance frequency in that ratio.

Reports of NAS of Ukraine [12] on the example of a famous problem of thin piezoceramic full electrode disk radial vibration the expressions for admittance were derived and graphs of the admittance change near resonance/anti-resonance at constant voltage (with constant amplitude) or constant current (with constant amplitude) were plotted. Calculations were made in complex form with accounting of experimentally determined mechanic, dielectric and piezoelectric energy losses. Calculated results were compared and agreed with experiment data. It was shown that for constant voltage regime instantaneous power sharply increases near resonance but it decreases for constant current regime. It explains existence or nonexistence of nonlinearity which was discovered in loss determination methodology for a piezoelectric ceramic [2].

This paper is devoted to analysis of the electric loading conditions problem for thin piezoceramic bars' forced vibrations. An idea of the step-by-step voltage measuring in modern Mason's schema [11,12] is continued and method of phase determination between admittance's

or power's components with cosine law is used [12-14]. New simple experimental technique together with computing permits to plot many resonators' parameters: admittance, impedance, phase shifts, power components etc for constant input voltage, constant sample voltage, constant sample current and constant instantaneous power electric conditions based on experimental data for "as there is" regime. Regimes constant voltage, constant current or constant power means that their amplitudes are installed at experiment beginning and not changed for all studied frequency range. Computer modeling makes possible to decrease of the experimental difficulties and to study in linear approximations the dependence of resonators' parameters from loading conditions.

It was established that admittance, impedance, phase shifts are independent from loading conditions. High admittance nonlinearity in constant voltage regime and its absence for constant current case may be created by different behavior of instantaneous power. To modeling constant power regime it is necessary to increase input voltage and sample voltage near resonance on 30% to 40% and in several times at anti-resonance in respect to off-resonance case.

Calculated and measured instantaneous power's spectrum for constant current conditions has the same view as impedance's spectrum while instantaneous power's spectrum for constant voltage has the same view as admittance's spectrum. Investigations show that constant current regime leads to decreasing resonant power and to increasing anti-resonant power, but constant voltage regime leads to increasing resonance power and to decreasing anti-resonance power.

Admittance, Impedance and Power Components for Thin Piezoceramic Bar

Piezoelectric bars with transversal polarization became already "touchstone" in many experimental researches because their vibrations

are described with simple mathematical formulas, and the first overtone lies far on frequency from fundamental resonance [1-3,9,10,15]. The mechanical and electric values of these structures are coupled between itself the so-called transversal coefficient of electromechanical coupling (EMC) k_{31}.

In Karlash, Shulga, and Uchino studies [11–14] it was shown that the piezoelectric resonator's admittance may be presented in calculations as imaginary conductivity of the inter-electrode sample capacity C_0 multiplied on anti-resonance $\Delta_a(x)$ to resonance $\Delta(x)$ determinants ratio:

$$Y = j\omega C_0 \frac{\Delta_a(x)}{\Delta(x)}, \tag{1}$$

Where x – is dimensionless frequency, which depends on geometric sample's form.

For thin bar with thickness polarization next formula was derived [16]

$$Y_b = j\omega C_0 \left[1 - k_{31}^2 + \frac{k_{31}^2 \sin x}{x \cos x}\right] = j\omega C_0 \frac{\Delta_a(x)}{\Delta(x)}, \tag{2}$$

$$\Delta(x) = \cos(x), \Delta_a(x) = (1 - k_{31}^2)\Delta(x) + k_{31}^2 \sin x / x.$$

Here: j is an imaginary unit, ω is an angular frequency, ω_r – resonant angular frequency, k_{31} – transverse coupling coefficient.

All variables, functions and electro-elastic coefficients are complex [4,11,16–18]. Dimensionless frequency x is definite as:

$$x = (\rho s_{11}^E)^{1/2} \omega l / 2 = (\rho s_{110}^E)^{1/2} \omega l / 2(1 - js_{11m}/2) = x_1 - jx_2, \tag{3}$$

$$(s_{11}^E = s_{110}^E(1 - js_{11m})).$$

Here: l is a bar's length, x_1, x_2 are real and imaginary parts of a complex dimensionless frequency, s_{11}^E – complex elastic compliances, ρ is a density.

Admittance (or full conductivity) characterizes sample's possibility to conduct electric current and determines as sample current to sample voltage ratio. This parameter is convenient near piezoelectric sample resonance where it reaches maximum value. But for anti-resonant frequency region more convenient is impedance because it reaches maximum value on anti-resonance. Admittance Y and impedance Z are in inverse ratio and for impedance calculation we may use a simple relation

$$Z = \frac{1}{Y}. \tag{4}$$

There are three characteristic power's components in sinusoidal alternative current circuit: instantaneous power P in volt-ampere (VA), which is voltage drop on some circuit area U in some time moment multiplied on circuit's current I in the same moment, active power P_a in Watts (W), which is distinguished on active resistance and heats it and reactive power P_{re} in volt-ampere reactive (VAr), which is distinguished on reactive circuit's elements (inductances or capacitance) and creates electric or magnetic fields

$$P = UI, P_a = UI \cos \alpha, P_{re} = UI \sin \alpha \cdot \tag{5}$$

When piezo resonator's voltage is U_1 and its current is I_0, we may write

$$U_1 = I_0 / Y, I_0 = U_1 Y, P = U_1 I_0 = I_0^2 / Y = U_1^2 Y, w_1 = \mathrm{Re}(Y) / \mathrm{Im}(Y),$$

$$\alpha = \mathrm{arc\, cot}(w_1), P_a = P \cos \alpha, P_{re} = P \sin \alpha. \tag{6}$$

Real part of coupling coefficient k_{31} may be determined in experiments with using formula [4,10,16]

$$\frac{k_{310}^2}{1 - k_{310}^2} = \frac{\pi}{2} \frac{f_n}{f_m} \tan\left[\frac{(f_n - f_m)\pi}{2f_m}\right] \tag{7}$$

Here: f_m and f_n are frequencies of maximum and minimum admittance respectively.

Real component of fundamental dimensionless resonance frequency for thin piezoceramic bar with thickness polarization is equal to $\pi/2$. Anti-resonance dimensionless frequency is dependent from coupling factor's value k_{31}, which is determined as [9,16].

$$k_{31}^2 = \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T} = \frac{d_{310}^2}{s_{110}^E \varepsilon_{330}^T}[1 + j(s_{11m} + \varepsilon_{33m} - 2d_{31m})] = k_{310}^2[1 + j(s_{11m} + \varepsilon_{33m} - 2d_{31m})], \tag{8}$$

$$(s_{11}^E = s_{110}^E(1 - js_{11m}), \varepsilon_{33}^T = \varepsilon_{330}(1 - j\varepsilon_{33m}), d_{31} = d_{310}(1 - jd_{31m}), d_{31}^2 = d_{310}^2(1 - 2jd_{31m})$$

In our experiments in parallel to generator's output or coordinating divider output was included a series circuit consisting from piezoelement Pe and loading resistors R [12,13]. We can measure for arbitrary chosen frequency range the voltages U_{pe} on the sample, U_R on loading resistor, input voltage U_{in}. An inter-electrode capacity C_0 and dielectric loss tangent tan = ε_{33m} are measured by alternative bridge at low frequency (1000 Hz). Voltage U_R is proportional to electrical current I_{pe} in resistor and sample.

$$I_{pe} = \frac{U_R}{R}. \tag{9}$$

The ratio of current to voltage is definite as admittance [6,13,17]

$$Y_{pe} = \frac{I_{pe}}{U_{pe}} = \frac{U_R}{R U_{pe}}. \tag{10}$$

Production of sample current on sample voltage is an instantaneous power

$$P_{pe} = U_{pe} I_{pe} = \frac{U_R U_{pe}}{R}. \tag{11}$$

Three measured voltages U_{pe}, U_R and U_{in} created peculiar characteristic triangle and angles between its sides were calculated with using a cosine low [12,14].

$$\cos \alpha = \frac{U_{pe}^2 + U_R^2 - U_{in}^2}{2U_{pe}U_R}, \cos \beta = \frac{U_{in}^2 + U_R^2 - U_{pe}^2}{2U_{in}U_R}, \cos \gamma = \frac{U_{in}^2 + U_{pe}^2 - U_R^2}{2U_{in}U_{pe}} \tag{12}$$

An angle α, formed sides U_R and U_{pe}, characterizes the change of phases between a current and voltage drop in piezoelement. An angle β, formed sides U_{in} and U_R, answers a phase shift between output voltage of generator and consumable current. And angle γ, formed sides U_{in} and U_{pe}, characterizes the difference of phases between output voltage of generator and voltage drop on piezoelement.

Voltages U_{pe}, U_R and U_{in} for "as there is" case together with according frequencies were entered in PC and admittance's, impedance's, angle's and power's AFChs were plotted with formulae (4), (9) – (12) using. To model constant current, constant voltage or constant power loading regimes the next simple transition formulae were derived

$$U_{in} = U_{in00}, U_{pe} = U_{in00}U_{pe0}/U_{in0}, U_R = U_{in00}U_{R0}/U_{in0};$$

$$U_R = U_{R00}, U_{in} = U_{R00}U_{in0}/U_{R0}, U_{pe} = U_{R00}U_{pe0}/U_{R0};$$

$$U_{pe} = U_{pe00}, U_{in} = U_{pe00}U_{in0}/U_{in0}, U_R = U_{pe00}U_{R0}/U_{pe0}; \tag{13}$$

$$p_0 = p_{00}, t = [p_0/(U_{R0}U_{pe0})]^{1/2}, U_{pe} = U_{pe0}t, U_R = U_{R0}t, U_{in} = U_{in0}t.$$

Here: U_{in00}, U_{pe00}, U_{R00} and p_{00} are constant amplitude values which are taken for modelling while U_{in0}, U_{pe0}, U_{R0} are ones for "as there is" regime.

Experimental-Calculation Methodology for Determination of the Coupling Factor and Loss Coefficients

It is necessary to know coupling and loss coefficients to calculate

any amplitude for piezoresonator's vibrations. I used the simple experimental-analytical iterative process, which permits to obtain the refined values based on experimental data [11-13]. Elaborated by me iterative methodology is that.

At first the resonance/anti-resonance frequencies and admittances are measured for first vibration mode as well as static capacity C_0 and dielectric loss tangent ε_{33m}. Values s_{11m}, ε_{33m}, k_{31}, d_{31m} enter to PC. Admittance AFCh in first resonance vicinity is plotted. A ratio maximum/minimum admittance's frequency is compared with measured one and k_{31} is correlated. Admittance AFCh in first resonance vicinity is plotted again and s_{11m} is correlated. When calculated values of Y_m and f_{a1}/f_{r1} coincide (or approach to) with measured ones it may be go to anti-resonant region and to find a piezoelectric loss tangent. Author's experience shows that good results are obtaining with three or four iterative steps. For first step any hypothetic data may be taken of cause [18].

Determinations of transversal EMCC (electro-mechanic coupling coefficient) k_{31}, and tangents of mechanical s_{11m} and piezoelectric d_{31m} losses of energy were performed by iterative steps for basic longitudinal resonance of bar with sizes 33.4 × 5.8 × 1.25 mm from piezoceramics TsTBS–3. It had C_0 = 2.98 nF, tanδ = ε_{33T} =0.0093, Y_m = 11.6 mS, Y_n = 0.0637 mS (Zn = 15.7 kOhm); f_m = 51.01 kHz; f_n = 53.04 kHz; f_n/f_m = 1.0398; a = 0.608 mS; x_{01} = 1.571. Multiplier a serves to coordinate dimensionless and resonant frequencies and it is introducing as

$$\omega C_0 = 2\pi f_{01} C_0 x / x_{01} = ax, \left(a = \frac{2\pi f_{01} C_0}{x_{01}} \right) \qquad (14)$$

Where x – running value of dimensionless complex frequency, x_{01} – active component of dimensionless frequency, f_{01} – measured frequency of admittance maximum.

The experimental measuring for thin piezoelectric bar fundamental mode of vibration was made with a loading resistor 229 Ohms.

Figure 1 illustrates three iterative steps (they were more in actual fact). For the sake of greater obviousness the plots of full conductivity are given to the first row, on the second row – entrance impedance. Plots for every step are similar and differ only with locations of a minimum of conductivity (or a maximum of impedance) on frequency and amplitudes of maximums and minimums.

As a result of the first iteration which was made in the interval of dimensionless frequencies 1.5 ≤ × ≤1.75 at the next chosen values k^2_{310} = 0.1; s_{11m} = 0.01; ε_{33m} = 0.0093; d_{31m} = 0.01; a = 0.608 mS such conductivities Y_m = 7.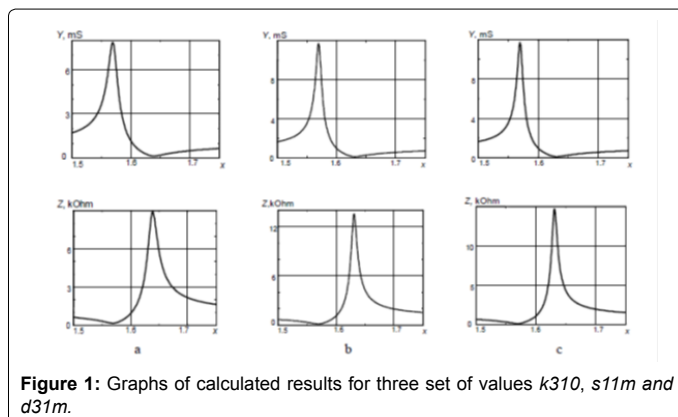8 mS; Y_n = 0.112 mS and frequencies x_n = 1.64; x_m = 1.571; x_n/x_m = 1.0446 were obtained (Figure 1a). It is need to diminish a ratio x_n/x_m, and it can be done only with diminishing of k^2_{310}, and for Y_m increasing it is necessary to diminish s_{11m}.

Next iteration was provided in the same frequency range at other set of values k^2_{310} = 0.09; s_{11m} = 0.006; ε_{33m} = 0.0093; d_{31m} = 0.007; a = 0.608 mS. Such conductivities Y_m = 11.7 mS; Y_n = 0.0735 mS and frequencies x_n = 1.6321; x_m = 1.571; x_n/x_m = 1.0389 were got (Figure 1b). Maximum of the complete conductivity and ratio of frequencies approach experimental values, and for diminishing of a minimum of conductivity it is needed to reduce the tangent of piezoelectric losses. Last iteration was conducted in a same frequency interval at d_{31m} = 0.004. It was got conductivities Y_m = 11.7 mS; Y_n = 0.0684 mS and frequencies x_n = 1.6321; x_m = 1.571; x_n/x_m = 1.0389 (Figure 1c). Disagreement with an experiment makes for Y_m – 0.86%, Y_n – 7.4%, f_n/f_m – 0.03%.

Thus, as a result of iterative procedures it is possible to consider set for the studying bar the following information: k^2_{310} =0.09; s_{11m} = 0.006; ε_{33m} = 0.0093 and d_{31m} = 0.004. This information may be used for calculations of admittances, impedances, powers, displacements, strains or stresses in any point of thin piezoelectric bar.

Similar iterative technique was used [18] for accurate determination of real and imaginary parts of the piezoceramics coefficients.

Modeling of an Electrical Loading Conditions of the Piezoelements

To estimate influence of the chosen electric loading regime on piezoceramic bar with dimensions 33.4 × 5.8 × 1.25 mm made of TsT-BC-3 ceramics forced vibrations amplitude-frequencies characteristics (AFChs) were plotted for the cases "as there is", constant sample current, constant sample voltage and constant sample power (Figure 2). A transition from one type of loading conditions to other is done with the use of formulae (13). There are plots in the first row "as there is", the second row answers constant current permanent amplitude of 1mA, the third row brings results over for the case of a voltage on piezoelement 100 mV (such level was experimentally got on resonance) and fourth row represents a case of constant instantaneous power level 0.218 mVA. The voltage drops U_{in} (solid lines) are given in the first column, U_{pe} (dashed curves) and U_R (dotted lines). Second and third columns are accordingly built for AFChs of full conductivity and instantaneous power. AFChs of phase angles α (solid lines), β (dotted lines) and γ (dashed curves) are resulted in a fourth column. Symbol "An" on the graphs means angle.
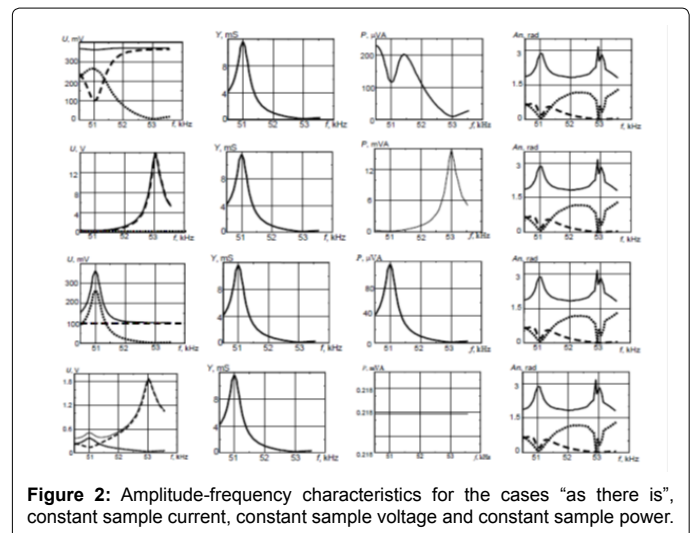

Figure 1: Graphs of calculated results for three set of values *k310, s11m and d31m.*

Figure 2: Amplitude-frequency characteristics for the cases "as there is", constant sample current, constant sample voltage and constant sample power.

The analysis of the graphs shows the next facts. The regime of the electric loading does not influence on AFChs of full conductivity and phase shifts – they remain without changes at the change of external electric conditions. At the same time, AFChs of instantaneous power and voltages strongly depend on the chosen loading regime. At approaching to resonance in the regime "as there is" a voltage U_{in} goes down (due to shunting operating of measuring circuit on the output of generator or coordinating voltage divider). Sharp growth of loading resistor U_R voltage takes a place and decline of voltage drops U_{pe} on piezoelement. And at nearing to anti-resonance the voltage drops on the loading resistor goes down and arrives at a minimum on certain frequency which is equated with anti-resonance [13,17,18].

An influence of electric loading conditions on piezoelectric ceramic sample vibrations' parameters differs for voltage drops, instant powers and admittance/impedance ones. Although piezoceramic materials are known as nonlinear substance author's experience and literature data testify that they may be regarded as liner for electric field up to1000 V/m level range. All graphs in this paper were built for linear approximations. The experimental investigation for thin piezoceramic disk radial vibrations at constant input voltage, constant sample voltage and constant sample current corroborated such intention [12].

Results and Discussion

Graphs of input U_{in} and sample U_{pe} voltages partly or fully coincide after transformations from "as there is" regime to constant current and constant power regimes. These curves do not be separated on black-white figs.

In regimes, constant sample current and constant sample voltage the frequency locations of instant power's maxima coincide with frequency locations of input voltage maxima. In regimes "as there is" the "failure" at resonance is observed in AFCh of instant power that may be result of part power reduction by loading resistor. It is necessary to increase of the input voltage at anti-resonance to realize of the constant sample current regime and at resonance for realization of the constant sample voltage regime. Regimes of constant instantaneous power may be realized with simultaneous increasing both resonant and anti-resonant input voltage. The degree of such increasing depend upon of loading resistor value and consists from a number of tens percents near resonance to several times near anti-resonance in respect to off-resonance case. The phase shift between sample voltage and sample current (angle α) reaches π radian at resonances and anti-resonances and it changes from π/2 to π in frequency range. The phase shift between output voltage of generator or voltage divider and consumable current (angle β) is equal to π/2 radian in frequency range [18]. It declines to zero at resonances and anti-resonances. And, difference of phases between output voltage of generator (voltage divider) and voltage drop on piezoelement (angle) declines to zero at resonances and may reach π/2 radian level at off-resonance frequencies.

Figure 3 demonstrates the experimental and transformed data for sample admittance and instantaneous sample power, obtained on PZT-type piezoceramic plate 40 × 16 × 1 mm with thickness polarization. Admittance spectrum is plotted for constant sample power (Figure 3a), constant current (Figure 3b), constant sample voltage (Figure 3c), and constant input voltage (Figure 3d) at frequency range below and near strong lateral resonance. Sample admittance is not dependent from electrical regime.

Power spectrum for regimes "as there is" (Figure 4a), constant current (Figure 4b), constant sample voltage (Figure 4c), and constant input voltage (Figure 4d) was built for same frequency range (Figure 4).
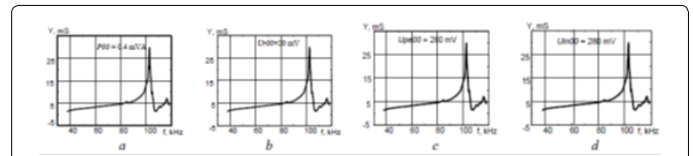

Figure 3: Transformed admittance spectrum for constant sample power (*a*), constant current (*b*), constant sample voltage (*c*), and constant input voltage (*d*) near strong lateral resonance.
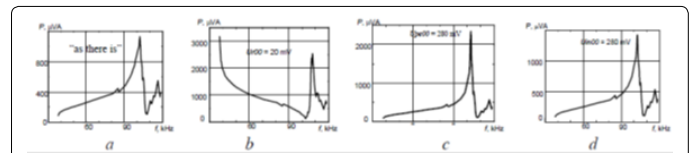

Figure 4: Transformed power spectrum for constant sample power (*a*), constant current (*b*), constant sample voltage (*c*), and constant input voltage (*d*) near strong lateral resonance.
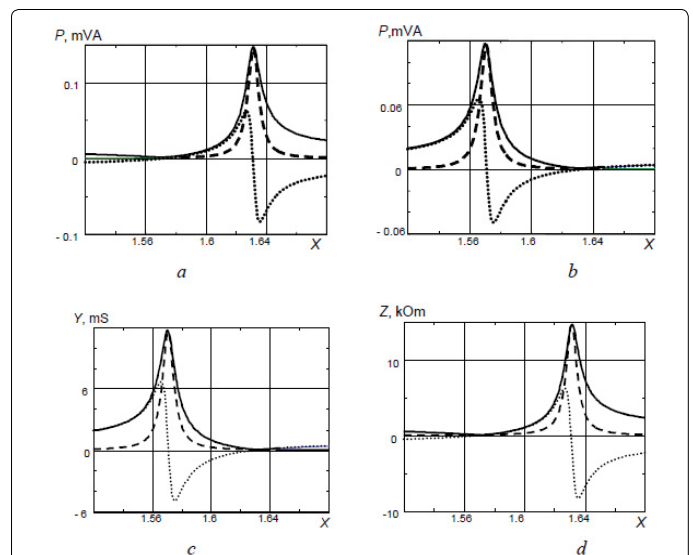

Figure 5: Calculated instantaneous power spectrum's view in comparison with admittance and impedance spectrums.

Constant current regime leads to decreasing resonant power and to increasing anti-resonant power, constant voltage and "as there is" regimes lead to increasing resonance power and to decreasing anti-resonance power. Such is on my mind a reason of nonlinearity, obtained by authors [2,15].

Formulae (5) and (6) were used for calculations of instantaneous power and its real or imaginary parts of thin piezoelectric bar's first longitudinal resonance. Calculations were made with next data k^2_{310} =0.09; $s_{11m} = 0.006$; $\varepsilon_{33m} = 0.0093$ and $d_{31m} = 0.004$ which were obtained as a result of iterative process for 33.4 × 5.8 × 1.25 mm bar from TsTBS–3 piezoceramics.

Figure 5 illustrates power frequency spectrum in range $1.52 \leq x \leq 1.68$ for constant current $I_0 = 0.1$ mA (a) and constant voltage $U_0 = 0.1$ V (b) loading conditions. The admittance for constant voltage (c) and impedance for constant current (d) AFCHs are plotted for compare. Full values of instantaneous power, admittance and impedance are given with solid lines. Their real parts are depicted as dashed curves and imaginary components are shown with dotted lines.

It is easy to see that instantaneous power spectrum's view for constant current conditions is the same as impedance's spectrum and

instantaneous power spectrum's view for constant voltage is the same as admittance's spectrum. Calculations show that constant current regime leads to decreasing resonant power and to increasing anti-resonant power, but constant voltage regime leads to increasing resonance power and to decreasing anti-resonance power.

Iterative process and linear approximation were used by me for more complicated cases such as circular segments or thin bars with divided electrode coating [18]. Results of these investigations are similar to results for thin bar vibrations case.

Conclusion

Amplitude-frequency characteristics for piezoelectric bars, disks, short and high cylindrical rings which were received in Institute Mechanics Research Laboratory (Department of Electro-elasticity) shown that electric loading regimes do not influence on admittance, impedance and phase angles. Independence these parameters from electrical loading regimes is explained simply – relations between voltages drops U_{in}, U_{pe} and U_R are not changed. But voltage drops and instantaneous power are very sensitive to loading conditions.

Results may be explained in such a way that when piezoelectric sample is excited by constant voltage the instantaneous power in sample increases at resonance frequency in many times in respect to off-resonant case. And when sample is excited by constant current the instantaneous power in sample decreases at resonance frequency in that ratio. Thus, the reason of admittance curve nonlinearity at constant voltage and its absence at constant current is upper or lower level of an instantaneous power.

A linear approximation for transformation of measured data permits to study main features of the piezoelectric resonators' vibrations without large mathematical and experimental difficulties. Such methodology gives possibility to make clear the general tendencies in piezoelectric sample behaviour for various electric loading conditions.

Instantaneous power spectrum's view for constant current conditions is the same as impedance's spectrum while instantaneous power spectrum's view for constant voltage is the same as admittance's spectrum. Calculations and experimental investigations show that constant current regime leads to decreasing resonant power and to increasing anti-resonant power, but constant voltage regime leads to increasing resonance power and to decreasing anti-resonance power.

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