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Commentary

The Theory of Single Track Vehicles

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ABSTRACT

Our paper arose from the desire to physically build a single track vehicle that would solve drawbacks of classic designs, while keeping all their advantages. Bringing such large task to success requires a strategy and, most of all, a method. The paper is mostly about the strategy of separation of concerns. We separate the theory, the design aspects and the construction. Our method is in being explicit about decisions that we make, and recording every step of our calculations. This first paper, of three, examines all that can be known about the motion of a single wheel, in a hope that its properties would carry over to vehicles with two or more wheels in line. The paper also lays down the notations and concepts of the theory as they are used in all three papers. The next paper deals with design of vehicles with multiple wheels in line. The final one deals with issues of construction.

Keywords: Single-track; Wheel; Stability; Precession; Slip; Counter-Steering

INTRODUCTION

We set out to design a better single track vehicle: a machine that is kept upright dynamically through the balancing action of its wheels. To accomplish our goal we developed a theory that would guide the designs to follow. First of all, we choose the simplest component that captures the most important characteristics of our subject, which is the wheel. Once we know every property of such a primitive of the theory, we may turn our attention to possibilities of combining multiple components together: the design.

Coordinate system

We aim to find out all about the balance, the motion and the direction. These sum up all we can say about a single wheel. All three are measured as angles, hence our coordinate system better be tri-polar. We ignore the concept of the distance, because we take the balance, motion and direction as immediate, at a given point. All plays out as if the wheel rotated, but still remained at the same contact point P.

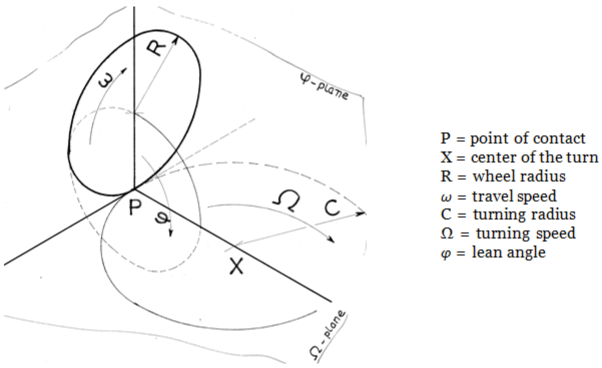

Figure 1: Tri-polar coordinate system.

Kinematic relations, balance and geometry

A wheel in motion follows naturally the trajectory of a conical roller:

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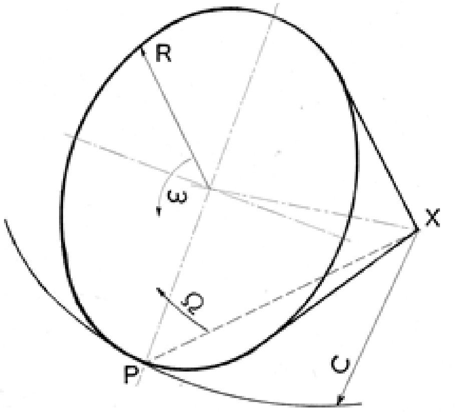

Figure 2: Conical roller.

The balance is a dynamic condition when the wheel travels along a constant trajectory with the constant lean angle. The forces brought about by the load m remain constant as well. In practise, the wheel either oscillates between the balance angles hardly ever reaching them:

they are the limits of the lean. Or, with a help of external moment -that counter-acts the righting moment- the wheel can be kept in balance at particular lean angle. Let γ=height of the center of gravity relative to wheel diameter.

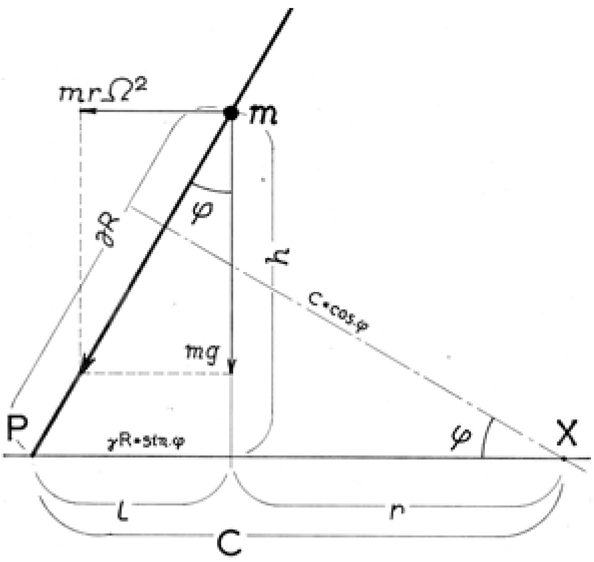

Figure 3: Balance.

Balance between centrifugal and gravity moments

The balance exists provided that the contact patch P offers the sufficient friction F against side sliding.

$F = mr\Omega^2$

$\Rightarrow$ { the moment pair exists }

$mr\Omega^2 * h = mg * l$

$\equiv$ { using (5) and /mh (dividing by m*h) }

$C * (1 - \gamma * \sin^2.\varphi) * \Omega = g * l/h$

$\equiv$ { using (3),(4) $l/h = R/(C * \cos.\varphi)$, shuffling }

$C * \Omega^2 = g * R/(C * \cos.\varphi * (1 - \gamma * \sin^2.\varphi))$

$\equiv$ { C to LHS, $v = C\Omega$ (wheel forward velocity) }

$v = g * R/(\cos.\varphi * (1 - \gamma * \sin^2.\varphi))$

Next, we graph the equation above, twice. Once for the wheel radius R=0.35m, with

γ=1, 2, 3, then for R=0.5m and γ=0.7, 1.4, 2.1. That gives us the same absolute heights of CG in both cases, but the radius of turns will not be the same. The larger wheel makes a larger turn at the same lean angle.

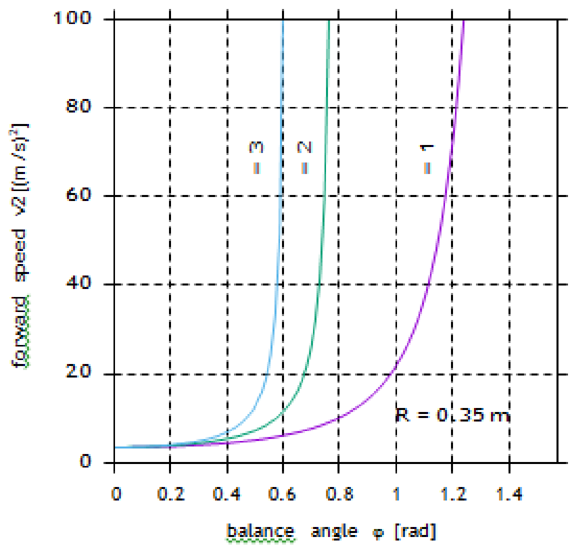

Figure 4: Balance angle.

We observe the paramount effect of γ on the ability to lean. When γ=3, which is the relative height of CG for a racing bicycle, the point of balance is about 0.58 rad (33°), at speed 10 m/s. However, one can place CG as low as γ=1 to achieve the balance at 1.1 rad (63°). Leaning past the point of balance colapses the wheel. To maintain a lesser, safe lean, there must be an external force counter-acting the wheel righting moment.

Righting moment

we identified two forces acting against each other over a lever. The resulting moment rightens the wheel up or collapses it down. In the following we follow the convention: The moment is positive when it increases the angle. Similarly for the angular velocity, it increases with the direction of angle.

$M.\varphi$
$=$ { using Fig.2 }
$mg * l - mr\Omega^2 * h$
$=$ { using (5) }
$m * (g * l - C * (1 - \gamma * sin^2.\varphi) * \Omega^2 * h)$
$=$ { $v = C\Omega$ }
$m * (g * l - (1 - \gamma * sin^2.\varphi) * v^2 * h/C)$
$=$ { using (4),(1) $h/C = \gamma * sin.\varphi * cos.\varphi$, $l = \gamma R * sin.\varphi$ }
$m * (g * \gamma R * sin.\varphi - (1 - \gamma * sin^2.\varphi) * v^2 * \gamma * sin.\varphi * cos.\varphi))$
$=$ { arithmetics, also $sin.x * cos.x = sin.2x/2$ }
$\gamma m * (gRsin.\varphi - v^2 * (sin.2\varphi/2 - \gamma sin^3.\varphi * cos.\varphi))$

The Specific Righting Moment is the Moment per 1 kg of load.

$$M^1.\varphi = \gamma(gRsin.\varphi - v^2 * (sin.2\varphi/2 - \gamma sin^3.\varphi * cos.\varphi))$$

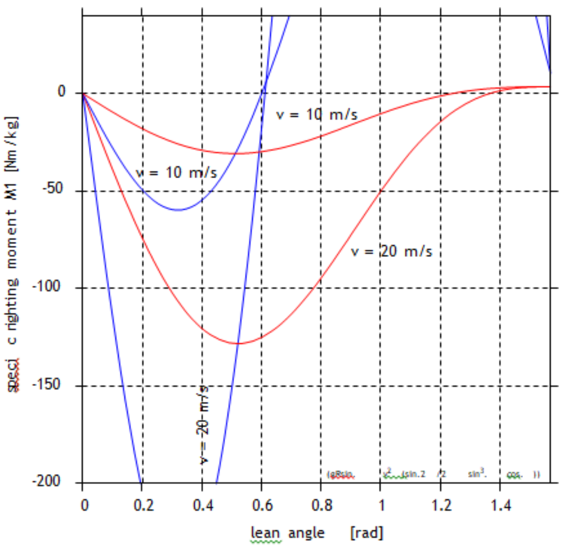

Figure 5: Righting moment for R=0.35m, γ=3 (blue), γ=1 (red).

The moment is negative when it decreases the φ. We observe that the righting moment peaks consistently at about 0.3rad (about 17°) for γ=3, and 0.5 rad (about 28°) for γ=1. After the peak angle, the controlled leaning of the wheel becomes difficult due to dropping feedbacks of the righting moment. We consider the peak the maximum safe lean angle.

Work to Lean

Does the steering and the balance have a cost? In other words, is there a loss of energy due to them? Let us find out how much work it takes to lean the wheel.

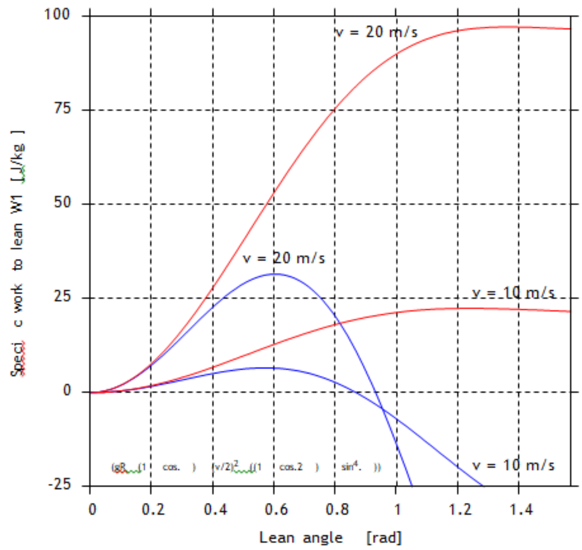

Figure 6: Work to lean R=0.35m, γ=3 (blue), γ=1 (red).

At small lean angles the amount of work to lean is nearly identical regardless of CG height. At large angles we see a dramatic difference between low and high CG when it comes to the amount of work, hence capacity to absorb disturbing impulses.

CONCLUSION

We formulated equations that capture the dynamics of the single wheel. In these equations we see the parameters that are the main players in the game of design, the subject of our next paper.