PREDICTION OF WAVE RUN-UP
ON A COASTAL IMPERMEABLE STRUCTURE

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ABSTRACT

A knowledge of wave run-up behavior on a coastal structure is one of the most important factors affecting the design of coastal structures exposed to wave attack. Prediction of wave run-up on a coastal impermeable structure with slope face and various friction factors are discussed with simplifying assumptions. An analytical approach is given for predicting wave run-up based on data measurements as the parameters of incident waves entering to coastal structure. Statistical approach with Weibull distribution is given on prediction of wave run-up, and present the probability distribution of wave run-up height.

I. INTRODUCTION

A knowledge of wave run-up behavior on a coastal structure is one of the most important factors affecting the design of coastal structures exposed to wave attack. The wave run-up on sloping faces is of considerable in the design of sea-walls and breakwaters. Over–topping of a protective structure can have most serious consequences, both in respect of damage to the structure itself and the flooding of areas on the land ward side.

In order to explain the wave run-up process, it is necessary to describe the characteristic of waves which waves undergo as they enter shallow water, break, and finally run up on the sloping face. Each of these stages is influenced by precedes it, so it is logical to start from the shallow water condition. As the waves entering into shallow water, they begin to be influenced by the presence of the bed when the water depth is equal to about half the water length. From this point both the speed of travel and wave length reduce, and also the wave height reduce slightly.

There has been considerable the experimental and theoretical interest in the approximates of wave run-up on sloping faces. Madsen and White (1976) have described a theoretical approach in the problem of energy dissipation on the seaward slope by a analyzing the associated of energy dissipation on a rough impermeable slope through an unknown wave friction factor. Van der Meer and Stam (1992) have derived qualitative analysis for wave run-up influenced by various parameters on run-up, and the formula for the assessment of various run-up levels as a function of the surf similarity or breaker parameter. Furthermore, Kobayashi and Wurjanto (1992) have extended the numerical model for predicting monochromatic or transient wave transformation and swash oscillation on a beach to predict normally incident random wave trans-formation in the surf and swash zones.

The main objective of this paper is to predict wave run-up on a coastal impermeable structure with slope face and various friction factors .based on periodic incident wave with simplifying assumptions. The method of the prediction is an analytical approach to derive wave run-up on a coastal impermeable structure, and to determine prediction of wave run-up height based on data measurements as the parameter of incident wave entering to the coastal structure. Furthermore, statistical approach with Weibul distribution is given for predicting incident wave height and wave run-up height, and present the probability distribution of wave run-up height. The data measurements is derived from the sea of coastal in Batang City with depth of water 5 meter.
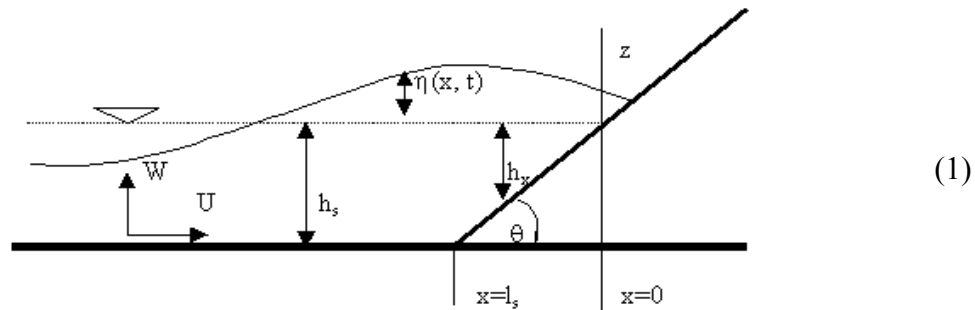

Figure 1. The Sketch of wave run-up on impermeable structure slope

II. WAVE RUN-UP EQUATION

Considered here the phenomena of wave run-up on coastal impermeable structure slope as in Figure 1. The wave run-up is describe on slope structure that are assumed to be impermeable slope with bottom friction, and on a bottom before the sloping face without friction. The depth of water before the sloping face is also a constant. Wave run-up is defined as the vertical height above still-water level to which a wave will rise on the slope structure. Waves are assumed to be relatively long waves of small amplitude normally incident on rough impermeable slope and are unbroken in the vicinity of the structure toe.

The parameters involved in describing wave run-up on impermeable structure slope is given in Figure 1. $\eta(x,t)$ is the free surface elevation above the still water level, U is the horizontal velocity component, h_x is constant depth of water before the sloping face, and h_x is the depth

assumed to very linearly corresponding to $h_x = x \ tn \ \theta$, and the slope θ is assumed relatively small. We assumed that the acceleration on terms are small and the elevation η is small compared with depth of water h_x. With these assumption, one dimensional wave run-up equation can be expressed in the form of two differential equations :

where $f_b = \frac{1}{2} f_w \ | \ U| \ / \ \omega h_x$, f_ω is a wave friction factor relating bottom shear tress, g is the acceleration due to gravity, ω is radian frequency of wave (Dean and Dalrymple, 1984). The wave run-up equation (1) is generated by the horizontal momentum

$$\left. \begin{array}{l} \dfrac{\partial \eta}{\partial t} + \dfrac{\partial(h_x U)}{\partial x} = 0 \\[2mm] \dfrac{\partial U}{\partial t} + f_b \ \omega \ U = - g \dfrac{\partial \eta}{\partial x} \end{array} \right\} \qquad (1)$$

equation (the first equation) and the linearized continuity equation for non

constant depth on slopping face h_x as a function of x and angle slope θ (the second equation). Differentiating the first equation of (1) with respect to time, then from second equation of (1), wave run-up equation can be expressed by the following

$$\frac{\partial^2 \eta}{\partial t^2} + f_b \, \omega \, \frac{\partial \eta}{\partial t} = g \frac{\partial}{\partial x}\left(h_x \frac{\partial \eta}{\partial x} \right) \quad (2)$$

To determine wave run-up on a coastal impermeable structures with slope face and friction factors from equation (1) and Figure 1, we suppose that the incident wave is expressed in the form of a periodic wave in terms of frequency ω. The free surface elevation of these incident wave can be expressed in the real part of the form

$$\eta(x,t) = \xi(x) \exp(i\omega t) \quad (3)$$

and their horizontal velocity component $U(x, t) = u(x) \exp(i\omega t)$, in which the complex amplitude function u and ξ as a function of any x .

For subdomain at $x \geq l_s$, we have assume that the bottom friction $f_b = 0$ and the depth of water before the sloping face is a constant in equation of (1) or (2). The general solutions for the governing partial differential equation (1) for subdomain at $x \geq l_s$ are

$\eta(x,t)=a_i \exp[i(kx+\omega t)]+a_r \exp[i(kx-\omega t)]$
$U(x,t)=-\sqrt{(g/h_s)}[a_I \exp[i(kx+\omega t)]$
$\qquad -a_r \exp[i(kx-\omega t)]$

where a_i and a_r are amplitude of incidence and reflected wave respectively (Graper, 1984), and k is number of wave, $k = 2\pi/L$, with L is the wave length in constant depth region. Then wave number k can be expressed by

$$k = \frac{\omega}{\sqrt{gh_s}} \quad (4)$$

The solution consists of an incident wave amplitude a_i and a reflected wave amplitude a_r.

Furthermore, for subdomain at $0 \leq x \leq l_s$ with $h_x = x \, tg \, \theta$, from wave run-up equation (2), we have differential equation as the following

$$x \frac{d^2\xi(x)}{dx^2} + \frac{d\xi(x)}{dx} + \frac{\omega^2 \, (1 - i \, f_b)}{g \, tg(\theta)} \xi(x) = 0$$

The solution of this equation can be expressed in the forms of Bessel functions. By this solution, remaining bounded at x = 0, the general solution of equation (1) as the wave run-up and horizontal velocity component of wave run-up can be expressed in the following forms

$$\eta(x,t) = AJ_o(F(x))\exp(i\omega t) \quad (5)$$

$$U(x,t) = -A\left[\frac{g}{(1 - i \, f_b) x \, tg(\theta x)}\right]^{\frac{1}{2}} \times J_1(F(x))\exp(i\omega t) \quad (6)$$

$$F(x) = 2\left(\frac{\omega^2(1 - f_b) \, x}{g \, tg(\theta t)}\right)^{\frac{1}{2}}$$

respectively, where A is arbitrary constant that is the vertical amplitude of the wave motion at the intersection of the still-water level and the slope x = 0, J_o is Bessel function of the first kind of order zero, and J_1 is Bessel function of the first kind of order one. The value of complex amplitude A in (5) expresses the amplitude of the vertical wave motion at the intersection of the slope and still water level x=0.The unknown amplitude A can be derive by matching the value of ξ and u at the common boundary x = l_s. Based on the simultaneous at their common boundary, amplitude A can be expressed by

The magnitude |A| does seem to predict the wave run-up on slope (Madson and White, 1991). By using this value of A, in the Section 4, we predict wave run-up height based on data measurements as the parameters of the incident waves.

$$\frac{A}{2a_i} = \frac{\exp(ikl_s)}{J_0(G) + \frac{i}{\sqrt{(1-if_b)}} J_1(G)} \quad (7)$$

$$G = 2kl_s \sqrt{(1-if_b)}$$

III. WAVE RUN-UP AS WEIBUL DISTRIBUTION

In this section, statistical approach with Weibul distribution is discussed for predicting incident wave height (amplitude) and wave run-up height, and present the probability distribution of wave run-up height. The cumulative Weibull distribution of significant wave height $\hat{H}_s$ particular value can be written as

$$P(H_s \le \hat{H}_s) = 1 - \exp\left[-\left(\frac{\hat{H}_s - B}{A}\right)^f\right]$$

where H_s is the significant wave height, f is shape parameter, A is scale parameter, and B is location parameter. Parameter A and B can be estimated by computing a lest squares fit based on the linear regression (Tucher 1991, Goda 1988) ;

$$H_{sm} = \hat{A} \, y_m + \hat{B} \quad (8)$$

The variable y_m is given by the following formulae

$$y_m = \left[-\{1 - \ln P(H_s \le H_{sm})\}\right]^{1/f} \quad (9)$$

H_{sm} is m^{th} value in the ranked significant heights, m is rank of the significant height value and the probability P is given by

$$P(H_s \le H_{sm}) = 1 - \frac{m - 0.2 - \frac{0.27}{\sqrt{f}}}{N_T + 0.2 + \frac{0.23}{\sqrt{f}}} \quad (10)$$

where m is rank of significant height value, and N_T is the total number of inputs significant heights. The sum of the squares of residual

$$\sum_{m=1}^{N_T} \left[H_{sm} - (\hat{A} \, y_m + \hat{B})\right]^2 \quad (11)$$

is selected to provide a good fit of the linear regression models (8).

IV. PREDICTION OF WAVE RUN-UP

In this section we present the prediction of wave run-up height on slopping faces with various friction factors based on data measurements as the parameters of incident wave. The sampling of wave data are derived from sea of coastal of Klidanglor, Batang at 25 October 2000 from 0:00 A.M. until 1:00 A.M. on 26 October 2000. The location of data measurements is about of 200 meter north from east breakwater with depth of water 5 meter. Figure 2 present the comparison of wave height and wave period associated from data measurements. In order to analyze wave run-up based on data observation as in Figure 2, we use the value of real part of amplitude of wave run-up A as in (5) with wave number k given in (4) to predict wave run-up height.

Figure 3 shows the computed prediction of run-up height based on wave run-up A with various friction factor f_b = 0,25; 0,50; 0,75 (Nur Yuwono, 1982). Based on amplitude of incident wave, the increasing average of wave run-up A is 149,93 % for friction factor f_b = 0,25, 97 % for f_b = 0,50, and 55,39 % f_b = 0,75.

To determine statistical prediction with Weibul distribution, first, compute the probability of the mth significant height not being exceeded as in (10) and the variable y_m in (9), then compute parameter A and B in the linear regression (8) by lest square

method. With associated shape parameter f = 0,75 (Goda, 1988), and from measurement of wave height as in Figure 2, the linear regression (8) can be expressed in the form

$$H_{sm} = 0,085 \ y_m + 0,1949$$

with the sum of the squares of residuals (11) is 0,065, and from the prediction of run-up A with friction factor f_b = 0,50 (Figure 3), the linear regression (8) can be expressed in the form

$$H_{sm} = 0,075 \ y_m + 0,1997$$

with the sum of the squares of residuals (11) is 0,092. The statistical prediction of amplitude of incident wave and wave run-up A with friction factor f_b = 0,50 are given in Figure 4 with probability distribution as in Figure 5.

Figure 5a. shows the incident wave height distribution, and Figure 5b. shows the wave run-up height distribution with friction factor f_b = 0,50. The value of probability is the probability of exceedance of particular value of significant wave height on significant wave height. From measurement data as in Figure 2, the mean of incident wave height is 0, 296 meter with probability of exceedance 0,320 and the significant of incident wave height (H_{33}) is 0, 455 meter with probability of exceedance 0,099. Furthermore, for wave run-up A with friction factor f_b = 0,50 as in Figure 4, the mean of wave run-up is 0, 289 meter with probability of exceedance 0,32 and the significant of incident wave height (H_{33}) is 0, 436 meter with probability of exceedance 0,094.
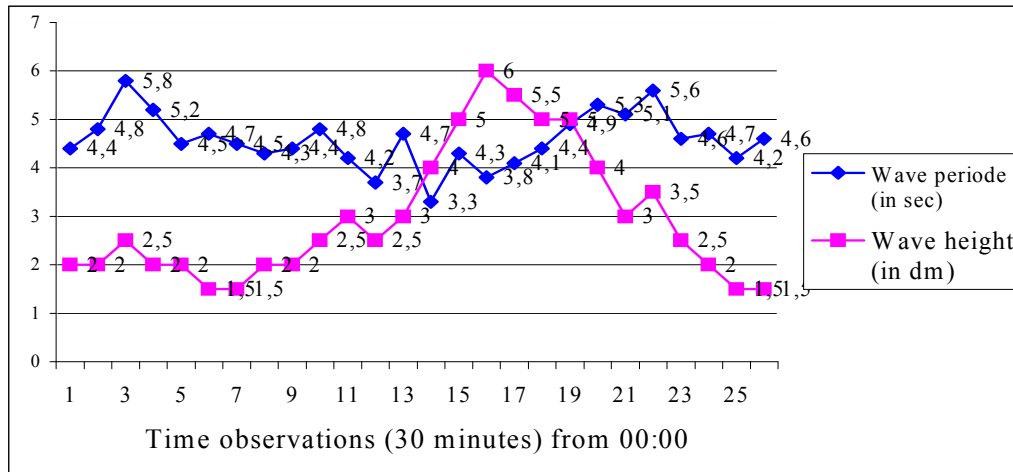

Figure 2. The comparison of wave height (in dm) and
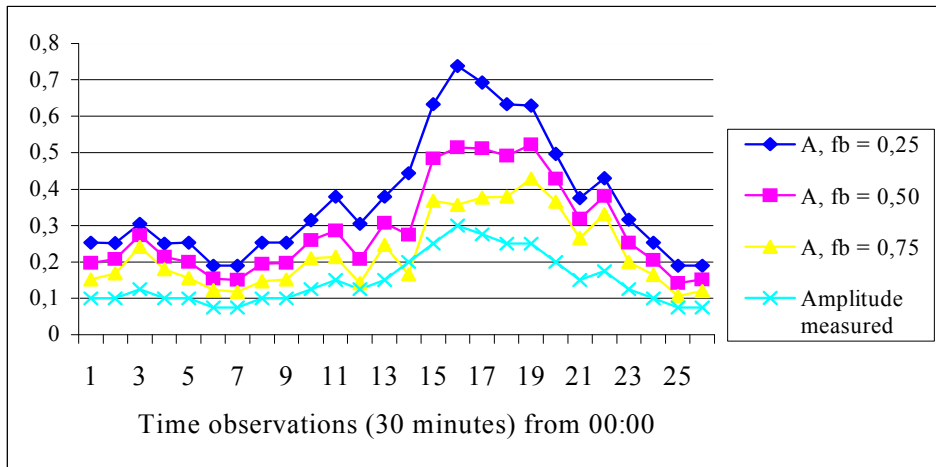wave period associated (in sec), water depth = 5 m.

Figure 3. The prediction of wave run-up A with various
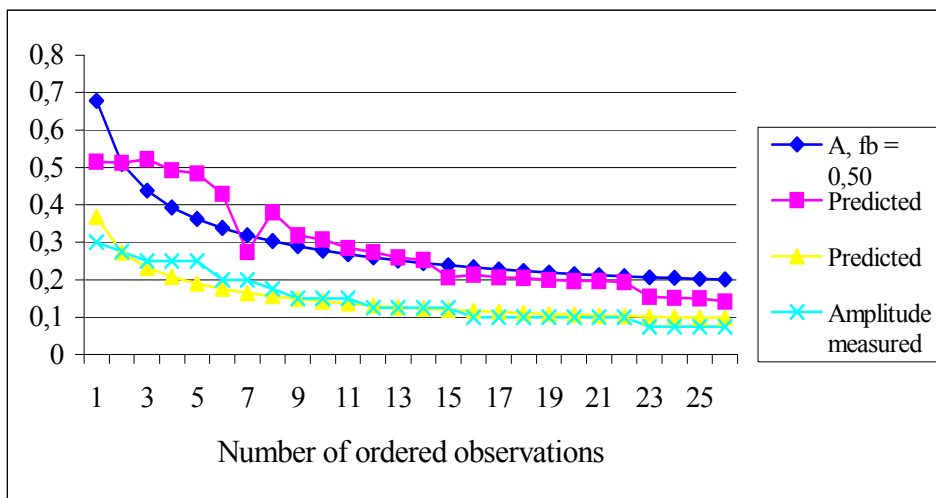friction factor (f_b = 0,25; 0,50; 0,75)

Figure 4. The statistical prediction of amplitude of
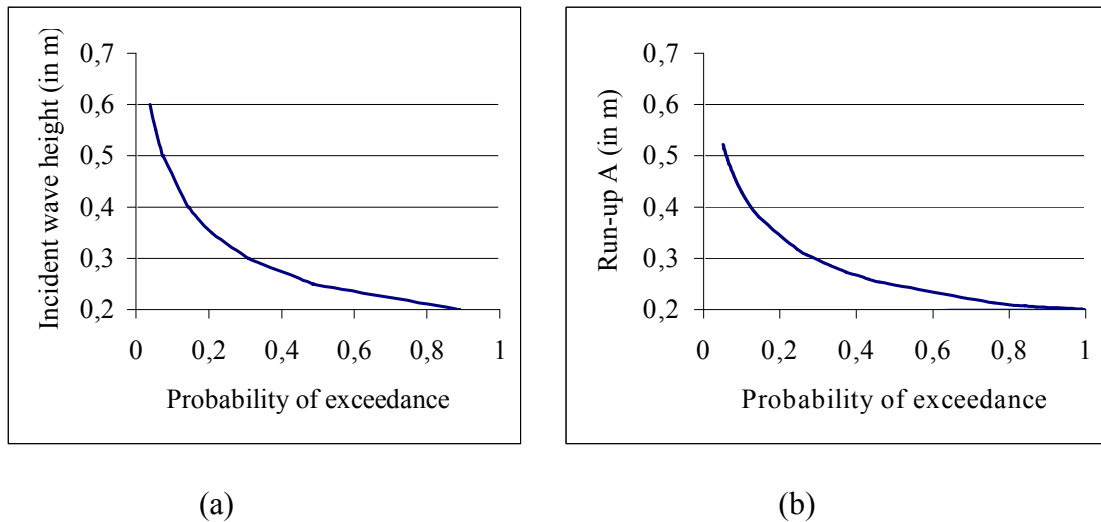incident wave and wave run-up (A) with
friction factor f_b = 0,50.

(a)   (b)

Figure 5. a. Incident wave height distribution,
                 b. Wave run-up A distribution (friction factor f_b =0,50)

V. CONCLUSIONS

The prediction of wave run-up is given on a coastal impermeable structure with slope face and various friction factors based on data measurements as the parameters of the incident wave. The formulation of wave run-up is expressed in term of Bessel function, with time independent complex amplitude A as the amplitude of the vertical wave motion at the intersection of the slope and still water level, x = 0. The prediction of wave run-up height based on wave run-up A with various friction factors are presented. Based on amplitude of incident wave, the increasing average of wave run-up height is 149,93 % for friction factor f_b = 0,25, 97 % for f_b = 0,50, and 55,39 % f_b = 0,75. The linear regression for predict the incident wave height and the wave run-up height is given with the sum of the squares of residuals as good fit of the models is 0,065 and 0,095 respectively. Furthermore, the probability distribution of the incident wave height and the wave run-up height are presented as a Weibul distribution.

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