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Original paper

NEW MODIFIED EQUATION OF LONGSHORE CURRENT VELOCITY AT THE BREAKING POINT (FOR MIXED AND GRAVEL BEACHES)

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ABSTRACT

Wave breaking is the dominant process in the dynamics of nearshore water movements resulting in sediment transport. The transformation of the subsequent particle motion from irrotational to rotational motion generates vorticity and turbulence and this affects the sediment transport. An improved understanding of the location of the breaker point and characteristics of the wave under these changing parameters is essential to our understanding of short and long-term morphological beach development.
This paper reports a series of 3-dimensional physical model tests to measure longshore current data, generated by oblique wave attack, along gravel and mixed beaches with a uniform slope and a trench. The studies described in this paper aim to improve the Longuet-Higgins's formulae which predicted the longshore current velocity at the breaking point.

Keywords: gravel, mixed beach, wave breaking, current velocity, longshore

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INTRODUCTION

Wave breaking at the shoreline is one of the least well understood of the coastal processes. There have been many stages and advances in our understanding of wave breaking, and these come predominantly from 2-dimensional physical model studies. To extend our understanding within the coastal environment a 3-dimensional physical model (see Figure 1Fig., Figure 2Fig., Table 1 and Table 2) was used to examine the longshore current velocity at the wave breaking for mixed and gravel beaches (Antoniadis, 2009).

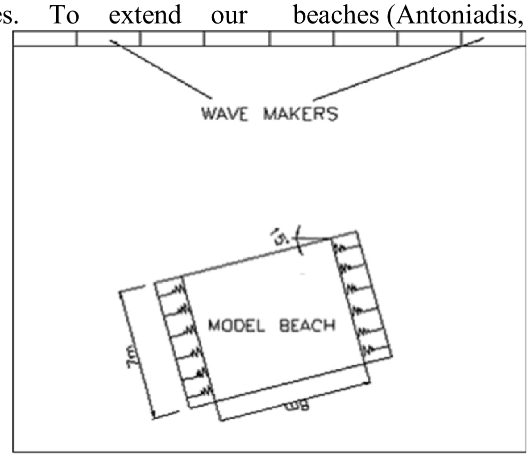

Fig.1 Position of the beach model

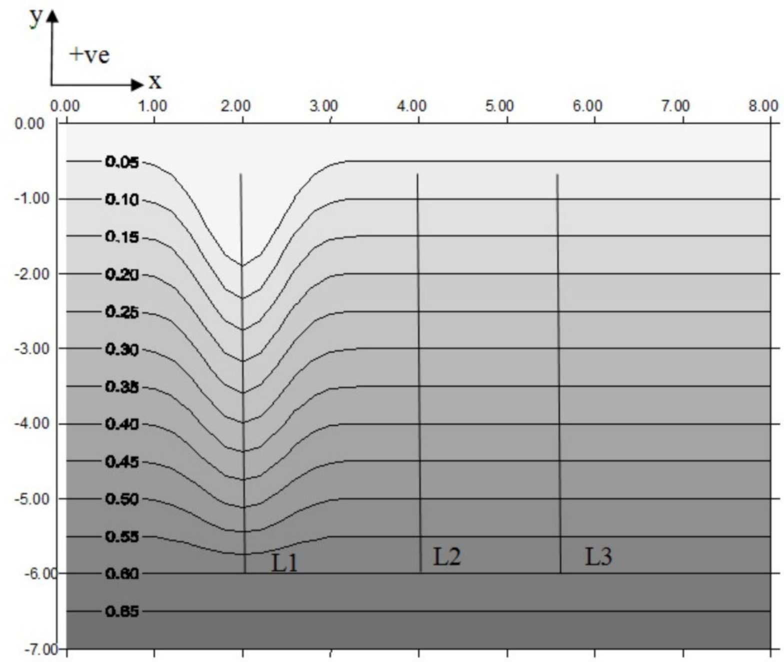

Fig.2 Model bathymetry (trench, uniform slope) and location of measurements

Table 1 The different particle sizes of the sediments

Type of Beach	D_5 (mm)	D_{15} (mm)	D_{16} (mm)	D_{50} (mm)	D_{84} (mm)	D_{85} (mm)	D_{90} (mm)	D_{94} (mm)
Gravel Beach (G)	15.35	16.66	16.83	22.76	28.38	28.86	29.59	30.50
Mixed Beach (M)	0.21	0.32	0.33	12	25.20	25.9	27.31	29.19

Table 2 Test programme of the experiments

TESTS (Regular Waves)	Wave Height (H)	Wave Period (T)	TESTS (Random Waves)	Significant Wave Height (H_{m0})	Spectral Peak Period (T_p)
Test 1-G	25.3 cm	2 sec	Test 5-G	10.8 cm	2.3 sec
Test 2-G	21.8 cm	3 sec	Test 6-G	11 cm	3.2 sec
Test 3-G	8.6 cm	2 sec	Test 9-M	11 cm	2.3 sec
Test 4-G	9.2 cm	3 sec	Test 10-M	11.7 cm	3.1 sec
Test 7-M	8.6 cm	2 sec			
Test 8-M	7.7 cm	3 sec			

Wave breaking depends on the nature of the bottom slope and the characteristics of the wave. Waves break as they reach a limiting steepness which is a function of the relative depth (d/L) and the beach slope (tanβ). Wave breaking may be classified in four types (Galvin 1968): as spilling, plunging, collapsing, and surging. Breaker type may be identified according to the surf similarity parameter (Iribarren number) ξ_0, defined as:

$$\xi_0 = \frac{\tan\beta}{\sqrt{\frac{H_o}{L_o}}}$$

(1)

where the subscript 0 denotes the deepwater condition (Galvin 1968, Battjes 1974). On a uniformly sloping beach, breaker type is estimated by:

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Surging/collapsing $\xi_0>3.3$
Plunging $0.5<\xi_0<3.3$ and,
Spilling $\xi_0<0.5$

Furthermore, the depth (d_B) and the height (H_B) of breaking waves are important factors. The term "breaker index" is used to describe non-dimensional breaker height. The four common indices are in the form of H_b/d_b, H_b/H_0, H_b/L_b and H_b/L_0 (where the subscript b denotes the breaking condition). The first two indices are the breaker depth index (γ) and the breaker height index (Ω_b), respectively.

Rattanapitikon and Shibayama (2000) examined the applicability of 24 existing formulas, for computing breaking wave height of regular wave, by wide range and large amount of published laboratory data (574 cases collected from 24 sources). They found that the formula of Komar and Gaughan (1973) gives the best prediction, among 24 existing formulas, over a wide range of experiments. Komar and Gaughan (1973) used linear wave theory to derive the breaker height formula from energy flux conservation and assumed a constant H_b/d_b. After calibrating the formula to the laboratory data of Iversen (1952), Galvin (1969), unpublished data of Komar and Simons (1968), and the field data of Munk (1949), the formula proposed was:

$$H_b = 0.56H_o\left(\frac{H_o'}{L_o}\right)^{-\frac{1}{5}} \text{ or } \Omega_b = 0.56\left(\frac{H_o'}{L_o}\right)^{-\frac{1}{5}}$$

(2)

where H_o' is the equivalent unrefracted deepwater wave height.

Rattanapitikon and Shibayama (2000) showed that the ER (root mean square relative error) of most formulae varies with the bottom slope, and it was expected that incorporating the new form of bottom slope effect into the formulas could improve the accuracy of the formulae. They therefore modified the three most accurate prediction formulae, concluding that the modified formula of Goda (1970) gives the best prediction for the general case (ER=10.7%).

The formula of Goda (1970) was modified to be:

$$H_b = 0.17L_0\left\{1-\exp\left[\frac{\pi d_b}{L_0}\left(16.21(\tan\beta)^2-7.07\tan\beta-1.55\right)\right]\right\}$$

(3)

The breaking depth, and consequently the breaking point, is also determined by using the Eq. (3) together with the linear wave theory. It is necessary that the breaking point is predicted accurately, in order for an accurate computation of the wave field or other wave-induced phenomena (e.g., undertow, sediment transport and beach deformation) to be concluded.

It is well known that the wave height, just before the breaking point, is underestimated by linear wave theory. Consequently, the predicted breaking point will shift on shoreward of the real one when the breaker height formula is used together with the linear wave shoaling (Isobe, 1987). As a result, the computation of wave height transformation will not be predicted accurately.

Two methods are known for dealing with the problem of underestimating the linear wave theory. The first method computes wave shoaling by using nonlinear wave theories (e.g. Stoke, 1847; Dean, 1965; Shuto, 1974; and Isobe, 1985) and the second method by using linear wave theory. The second method also uses other variables, rather than breaker height, to compute the breaking point (e.g. Watanabe et al., 1984; Isobe, 1987; Rattanapitikon, 1995 and Rattanapitikon and Shibayama, 2006).

Rattanapitikon and Shibayama (2006), by following the second method, undertook a study to find out the suitable breaking wave formulas for computing breaker depth, and corresponding assumed orbital to phase velocity ratio and breaker height converted with linear wave theory. A total of 695 cases collected from 26 sources of published laboratory data were used. All data referred to experiments that were performed on regular waves. The formulae of Rattanapitikon and Shibayama (2006) gave satisfactory predictions over a wide range of experimental conditions. Their formulae for breaking depth and breaking wave height were:

$$h_b = (3.86m^2 - 1.98m + 0.88)H_0\left(\frac{H_0}{L_0}\right)^{-0.16} \text{ for } \frac{H_0}{L_0} \leq 0.1$$

(4a)

$$h_b = (3.86m^2 - 1.98m + 0.88)H_0\left(\frac{H_0}{L_0}\right)^{-0.34} \quad \text{for } \frac{H_0}{L_0} > 0.1$$

(4b)

and

$$H_b = (-0.57m^2 + 0.31m + 0.58)L_0\left(\frac{H_0}{L_0}\right)^{0.83}$$

(5)

where m is the bed slope.

Random waves consist of incoming waves which have different wave height and they break in different water depths. Therefore, the wave breaking takes place in a relatively wide zone (surf zone) of variable water depth. Goda's breaking method (Goda, 1985) is the most widely applied method for estimating significant wave heights ($H_{1/3}$) within the surf zone. Goda (1970) proposed a diagram, presenting criterion for predicting breaking wave height, based on the analysis of several sets of laboratory data of breaking waves on slopes obtained by several researchers (Iversen, 1952; Mitsuyasu, 1962; and Goda, 1964). Goda gave an approximate expression of the diagram as

$$H_b = AL_0\left\{1 - \exp\left[-1.5\frac{\pi d_b}{L_0}\left(1 + 15(\tan\beta)^{\frac{4}{3}}\right)\right]\right\}$$

(6)

where A= a coefficient (=0.12)

The breaking point is defined as the maximum wave height admissible for a given water depth (Torrini and Allsop, 1999).

Materials and Methods

The breaking of obliquely waves generates currents which usually dominate in and near the surf zone on open coasts. These wave driven currents have long-shore and cross-shore components. In this section, the long-shore velocity (v_B) at the breaking point has been calculated in order to be compared with the results of the experimental tests for both gravel and mixed beaches.

For the theoretical approximation of the v_b the wave refraction and shoaling were included. Moreover, the seabed contours were assumed to be straight and parallel for both trench and beach with uniform slope. Despite the fact that trench usually does not have straight and parallel contour, this assumption was adopted. Moreover, approaches and equations that derived for planar beach, in their original form, were applied also at the trench. However, these approaches and equations, used for trench, were modified in order the effect of the complex sea bed contour to be reduced as more as possible.

Results and Discussion

Regular Waves

The following procedure relates to the estimation of breaking wave height and depth and is applied to regular waves. The deep water wavelength and celerity are calculated by:

$$L_0 = \frac{C_0}{T}$$

(7)

$$C_0 = \frac{gT}{2\pi}$$

(8)

the water wavelength by,

$$= \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$$

(9)

The shoaling coefficient K_S and refraction coefficient K_R can be estimated from,

$$K_S = \left(\frac{C_0}{C\left[\frac{2kd}{\sinh 2kd}\right]}\right)^{\frac{1}{2}}$$

(10)

and

$$K_R = \left(\frac{\cos\theta_0}{\cos\theta}\right)^{\frac{1}{2}}$$

(11)

where θ_0 is the deepwater wave angle, where the wave number k is equal to $2\pi/L$.

Assuming that a refraction analysis gives a refraction coefficient K_R at the point where breaking is expected to occur, and that the equivalent unrefracted deepwater wave height can be found from the refraction coefficient

$H_o' = K_R H_0$, consequently

$H = H_o' K_S$

(12)

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Then by estimating the breaking wave height, the breaking depth can be calculated by corresponding equation.

The initial value selected for the refraction coefficient would be checked to determine if it is correct for the actual breaker location. If necessary, a corrected refraction coefficient should be used to recompute the breaking wave height and depth.

Longuet-Higgins (1970) formed an expression for the mean longshore velocity ($\bar{v}_l$) at the breaker zone, of a planar beach, which was modified by Komar (1976) and took the form of:

$$\bar{v}_l = 2.7 u_b \sin\theta_b \cos\theta_b \quad (13)$$

where θ_b= the wave angle at the breaking point

u_b= the wave orbital velocity under the wave breaking point, which is calculated by

$$u_b = \frac{\gamma}{2}\sqrt{g d_B} \quad (14)$$

where γ= breaking depth index (H_b/d_b)

Longuet-Higgins (1972) stated that the longshore velocity at the breaking point (v_B) is usually about $0.2\bar{v}_l$. Therefore, knowing the breaking depth and height, the longshore velocity at the breaking point can be estimated by

$$v_B = 0.54 \frac{\gamma}{2}\sqrt{g d_B} \sin\theta_b \cos\theta_b \quad (15)$$

Moreover for a plane beach where d = xtanβ (tanβ is the beach slope), the distance to the breaker line from shore is

$$x_B = \frac{d_B}{\tan\beta} \quad (16)$$

Using the above equations, v_B was calculated for all the tests with regular waves. The slope between Lines 2 and 3 (Figure 2) was approximately the same. Test 2 wasn't taken into consideration for the calculations due to the fact that the slope changed significantly after Test 1. However, Eq. (14) was not based on a wave breaking equation that includes the influence of the slope. Therefore, the three lines will be considered as one. The wave conditions for both gravel and mixed beaches were not exactly the same (except the Tests with wave height H=0.086m). Consequently, the longshore velocity at the breaking point would be similar for both types of beach, only in Tests 3 and 7. The results of the calculations are shown in Table 3Table 3.

Table 3 The results of the calculations of v_B for the tests with regular waves

Test (No.)	**H (m)**	**T (sec)**	**Θ (^{0})**	**d_B (m)**	**v_B (cm/s)**
1	0.253 (G)	2	15	0.326	5.20
3	0.086 (G)	2	15	0.132	2.19
4	0.092 (G)	3	15	0.161	1.81
7	0.086 (M)	2	15	0.132	2.19
8	0.077 (M)	3	15	0.139	1.57

It has to be mentioned that the equation of Longuet-Higgins (1972) did not take into consideration the spatial and temporal variability. The beach profile of each line has been changed through time due to the sediment transport. Therefore, the break point of each line changed and consequently v_B changed. However, for the purpose of the comparison and the analysis of the equation of Longuet-Higgins (1972), it was assumed that there were not any spatial and temporal variability.

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 4 and Table 5. It has to be mentioned that when the column of measured v_B had negative values, it meant that the longshore current velocity was in opposite direction with the incoming wave direction and where the column has no number, it meant that there were no measurements (or measurements with less than 70% correlation) at that point.

Table 4 The measured and estimated v_B at the tests with gravel beach

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
1	0.253 (L1)	2	0.326	5.20	2.36
1	0.253 (L2)	2	0.326	5.20	-4.85
1	0.253 (L3)	2	0.326	5.20	-6.52
3	0.086 (L1)	2	0.132	2.19	2.51
3	0.086 (L2)	2	0.132	2.19	7.45
3	0.086 (L3)	2	0.132	2.19	12.65
4	0.092 (L1)	3	0.161	1.81	-2.41
4	0.092 (L2)	3	0.161	1.81	0.26
4	0.092 (L3)	3	0.161	1.81	-

Table 5 The measured and estimated v_B at the tests for the mixed beach

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
7	0.086 (L1)	2	0.131	2.18	-
7	0.086 (L2)	2	0.131	2.18	9.19
7	0.086 (L3)	2	0.131	2.18	-
8	0.077 (L1)	3	0.139	1.57	-
8	0.077 (L2)	3	0.139	1.57	-
8	0.077 (L3)	3	0.139	1.57	10.13

The breaking longshore velocity has been chosen based on the value of the estimated breaking depth. It must be mentioned that the accuracy of the measurements of the ADV was ±0.5%.

Looking at Table 4 and Table 5, the estimated v_B from Longuet-Higgins (1972) equation did not predict accurate results. Generally, it underestimated the measured v_B. At some tests/lines the estimated v_B was 9 times greater than the measured v_B and at some other it was 7 times smaller. The estimated v_B was similar to the measured v_B, only in Tests 1, 3 and 4 (especially for Line 1). At these tests, the magnitude of the v_B was similar but not its direction. At the tests related to the mixed beach, there were only few available locations to compare with. Based on the theory that the longshore velocity at the breaking point would be the same for both types of beach, if both types of beach have the same wave conditions, the measured longshore velocity at the breaking point for Line 3 gave similar values for both types of beach for Tests 3 and 7. However, based on the assumption that the estimated breaking depth was accurate, it can be seen that the measured longshore "breaking" velocity had different values for all three lines.

This happened due to the fact that the estimated v_B of Longuet-Higgins (1972) was based on a wave breaking equation that did not take into consideration the influence of the bottom slope (H_d=0.78d_b). Therefore, in order to include the influence of the bottom slope, the estimated breaking depth of Eq. (3) were used into Eq. (14). The longshore "breaking" velocities of Lines 2 and 3 were calculated as one due to the fact that the bottom slopes of both lines were approximately the same.

At Line 1, where the trench was, the calculation of the breaking depth and consequently of v_B based on different bottom slope from the other two Lines. The trench had two bottom slopes. The first slope was nearly horizontal. Based on the wave conditions in the tests, the first slope wouldn't affect the breaking depth and breaking height. Therefore, the second bottom slope has been used for the calculation of d_B. As previously, Test 2 wasn't considered in the calculations due to the fact that the bottom slope changed significantly

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after Test 1. The results of the calculations for Lines 2 and 3 are shown in Table 6

Table 6 The results of the calculations of v_B for the tests with regular waves (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	θ (^{0})	ξ	d_B (m)	v_B (cm/s)
1	0.253 (G)	2	15	0.55	0.266	5.45
3	0.086 (G)	2	15	0.85	0.102	2.32
4	0.092 (G)	3	15	1.11	0.125	1.93
7	0.086 (M)	2	15	0.85	0.102	2.32
8	0.077 (M)	3	15	1.22	0.108	1.67

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 7 and Table 8.

Table 7 The measured v_B at the tests with gravel beach (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
1	0.253 (L2)	2	0.266	5.45	-
1	0.253 (L3)	2	0.266	5.45	-3.54
3	0.086 (L2)	2	0.104	2.31	-
3	0.086 (L3)	2	0.104	2.31	-
4	0.092 (L2)	3	0.125	1.93	6.31
4	0.092 (L3)	3	0.125	1.93	-

Table 8 The measured v_B at the tests with mixed beach (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
7	0.086 (L2)	2	0.102	2.32	-
7	0.086 (L3)	2	0.102	2.32	-
8	0.077 (L2)	3	0.108	1.67	-
8	0.077 (L3)	3	0.108	1.67	-

The results of the calculations for Line 1 are shown in Table 9.

Table 9 The results for the calculations of v_B for the tests with regular waves (Line 1)

Test (No.)	H (m)	T (sec)	θ (^{0})	ξ	d_B (m)	v_B (cm/s)
1	0.253 (G)	2	15	0.65	0.259	5.48
3	0.086 (G)	2	15	0.85	0.102	2.32
4	0.092 (G)	3	15	1.48	0.120	1.95
7	0.086 (M)	2	15	0.85	0.102	2.32
8	0.077 (M)	3	15	1.35	0.106	1.68

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 10 and Table 11.

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Table 10. The measured v_B at the tests with gravel beach (Line 1)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
1	0.253	2	0.259	5.48	-
3	0.086	2	0.102	2.32	-
4	0.092	3	0.120	1.95	-

Table 10 The measured v_B at the tests with mixed beach (Line 1)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
7	0.086	2	0.102	2.32	-
8	0.077	3	0.106	1.68	-

Despite the fact that the new estimated v_B had few available locations to compare with, especially for tests with mixed beach where there were not any measurements at these breaking depths for both trench and uniform slope, it gave slightly better results than the previous estimated v_B of Longuet-Higgins equation. There were not any available measurements for trench for both types of beach. In general, the estimated value of v_B was still not close enough to the measured v_B.

Rattanapitikon and Shibayama (2006) undertook a study to find out the suitable breaking wave formulas for computing breaker depth, and corresponding orbital to phase velocity ratio and breaker height converted with linear wave theory.

With regard to assumed orbital to phase velocity, only the formula of Isobe (1987) was available. Rattanapitikon and Shibayama (2006) developed a new formula by reanalysis of the Isobe's (1987) formula. The new formula gave excellent predictions for all conditions (ER_{avg}=3%). The assumed orbital velocity ($\widehat{u_b}$) formula of Rattanapitikon and Shibayama (2006) was written as:

$$\widehat{u_b} = \frac{(-0.57m^2+0.31m+0.58)\pi c_b}{\tanh^2(k_b h_b)}\left(\frac{H_c}{L_c}\right)^{0.83} \quad (17)$$

where,
c_b is the phase velocity at the breaking point, k_b is the wave number at the breaking point, m is the bottom slope and h_b is the breaker depth (Eq. 5). Eq. (14) was substituted by Eq.(17) in the Longuet-Higgins's (1972) equation. The new equation has the form of:

$$\overline{v_l} = 2.7\widehat{u_b}\sin\theta_b\cos\theta_b \quad (18)$$

and consequently,

$$v_B = 0.54\widehat{u_b}\sin\theta_b\cos\theta_b \quad (19)$$

The results of the calculations, by using Eq. (17) and Eq. (19), for Lines 2 and 3 are shown in Table 12.

Table 11 The results for the calculations of v_B for the tests with regular waves (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	θ (^{0})	ξ	d_B (m)	$\hat{u}_b$ (m/s)	v_B (cm/s)
1	0.253 (G)	2	15	0.55	0.301	0.841	6.04
3	0.086 (G)	2	15	0.85	0.123	0.502	2.39
4	0.092 (G)	3	15	1.11	0.151	0.539	1.92
7	0.086 (M)	2	15	0.85	0.123	0.502	2.39
8	0.077 (M)	3	15	1.22	0.130	0.498	1.65

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 13 and Table 14.

Table 12 The measured v_B at the tests with gravel beach (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
1	0.253 (L2)	2	0.300	6.04	1.25
1	0.253 (L3)	2	0.300	6.04	-6.29
3	0.086 (L2)	2	0.125	2.39	4.31
3	0.086 (L3)	2	0.125	2.39	12.65
4	0.092 (L2)	3	0.151	1.92	0.59
4	0.092 (L3)	3	0.151	1.92	-

Table 13 The measured v_B at the tests with mixed beach (Line 2 and Line 3)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
7	0.086 (L2)	2	0.123	2.39	-
7	0.086 (L3)	2	0.123	2.39	11.86
8	0.077 (L2)	3	0.130	1.65	-
8	0.077 (L3)	3	0.130	1.65	-

The results of the calculations, by using equations Eq. (18) and Eq. (19), for Line 1 are shown in Table 15.

Table 14 The results for the calculations of v_B for the tests with regular waves (Line 1)

Test (No.)	H (m)	T (sec)	θ (^{0})	ξ	d_B (m)	$\hat{u}_b$ (m/s)	v_B (cm/s)
1	0.253 (G)	2	15	0.65	0.292	0.856	6.07
3	0.086 (G)	2	15	0.85	0.123	0.502	2.39
4	0.092 (G)	3	15	1.48	0.144	0.557	1.94
7	0.086 (M)	2	15	0.85	0.123	0.502	2.39
8	0.077 (M)	3	15	1.35	0.127	0.504	1.66

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 16Table 15 and Table 17.

Table 15 The measured v_B at the tests with gravel beach (Line 1)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
1	0.253 (L1)	2	0.291	6.07	7.95
3	0.086 (L1)	2	0.119	2.41	-1.86
4	0.092 (L1)	3	0.144	1.94	-

Table 16 The measured v_B at the tests with mixed beach (Line 1)

Test (No.)	H (m)	T (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
7	0.086 (L1)	2	0.123	2.39	-
8	0.077 (L1)	3	0.128	1.66	-

The values of estimated v_B were close to the values of measured v_B for Line 1 (for both types of beach) and for Line 3 (for gravel beach). It estimated quite accurately the magnitude of the v_B for few tests. However, it also underestimated, as in the previous approaches, the value of v_B in some occasions.

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Overall, Eq. (19) gave much more accurate results than the previous equations.

Based on the experimental results and results of Eq. (19), two equations are proposed for estimation of the mean longshore velocity at the breaking point. A linear regression has been fitted to the data and the proposed fits are given by the following equations:

For gravel beach-trench,

$$v_B = 0.554\widehat{u_b}\sin\theta_b\cos\theta_b$$

(20a)

For mixed beach-uniform slope

$$v_B = 2.68\widehat{u_b}\sin\theta_b\cos\theta_b$$

(20b)

Random Waves

The procedure of estimating the breaking wave height and depth for random waves is described in Appendix A. In this section, Eq. (19) was used to estimate the mean long-shore current at the breaking point as it was the most accurate equation for regular waves. However, the breaking depth will not be calculated by Eq. (4) but with Eq. (6).

The results of the calculations, by using Eq. (19) with Eq. (6), for Lines 2 and 3 are shown in Table 18.

Table 17 The results for the calculations of v_B for the tests with random waves (Line 2 and Line 3)

Test (No.)	H (m)	T_s (sec)	θ (^{0})	ξ	d_B (m)	$\hat{u}_b$ (m/s)	v_B (cm/s)
5	0.108 (G)	2.26	15	0.77	0.183	0.696	3.72
6	0.110 (G)	3.24	15	1.10	0.222	0.852	3.53
9	0.110 (M)	2.28	15	0.86	0.179	0.724	3.81
10	0.117 (M)	3.05	15	1.45	0.200	0.964	4.03

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 19 and Table 20.

Table 18 The measured v_B at the tests with gravel beach (Line 2 and Line 3)

Test (No.)	H (m)	T_s (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
5	0.108 (L2)	2.264	0.183	3.72	3.63
5	0.108 (L3)	2.264	0.183	3.72	2.04
6	0.110 (L2)	3.244	0.222	3.53	3.03
6	0.110 (L3)	3.244	0.222	3.53	3.05

Table 19 The measured v_B at the tests with mixed beach (Line 2 and Line 3)

Test (No.)	H (m)	T_s (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
9	0.110 (L2)	2.278	0.179	3.81	-
9	0.110 (L3)	2.278	0.179	3.81	-
10	0.117 (L2)	3.053	0.200	4.03	1.21
10	0.117 (L3)	3.053	0.200	4.03	1.95

The results of the calculations, by using Eq. (19) with Eq. (6), for Line1 are shown in Table 21.

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Table 20 The results for the calculations of v_B for the tests with random waves (Line 1)

Test (No.)	H (m)	T_s (sec)	θ (^{0})	ξ	d_B (m)	$\hat{u}_b$ (m/s)	v_B (cm/s)
5	0.108 (G)	2.264	15	0.95	0.172	0.746	3.87
6	0.110 (G)	3.244	15	1.46	0.203	0.947	3.76
9	0.110 (M)	2.278	15	0.94	0.174	0.750	3.89
10	0.117 (M)	3.053	15	1.67	0.190	1.03	4.19

In order to compare the estimated values of v_B with the measured v_B from experimental results (for both types of beach), the data have been tabulated and presented in Table 22 and Table 23.

Table 21 The measured v_B at the tests with gravel beach (Line 1)

Test (No.)	H (m)	T_s (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
5	0.108 (L1)	2.264	0.172	3.87	2.58
6	0.110 (L1)	3.244	0.203	3.76	3.25

Table 22 The measured v_B at the tests with mixed beach (Line 1)

Test (No.)	H (m)	T_s (sec)	d_B (m)	v_B (cm/s) estimated	v_B (cm/s) measured
9	0.110 (L1)	2.278	0.174	3.89	-
10	0.117 (L1)	3.053	0.190	4.19	-2.90

It can be seen that Eq. (19) gave satisfactory results for gravel beach. The v_B was often overestimated for mixed beach. Based on the present experimental results and results of Eq. (19), three equations are proposed for the mean longshore velocity at the breaking point for random waves. A linear regression has been fitted to the data and the proposed fit is given by the following equation:

For gravel beach-uniform slope

$$v_B - 0.438\hat{u}_b \sin\theta_b \cos\theta_b \quad (21a)$$

For mixed beach-uniform slope

$$v_B = 0.212\hat{u}_b \sin\theta_b \cos\theta_b \quad (21b)$$

For gravel beach-trench

$$v_B = 0.412\hat{u}_b \sin\theta_b \cos\theta_b \quad (21c)$$

Conclusion

This paper introduced an improvement on the equation derived by Longuet-Higgins (1970), and modified by Komar (1976), in order to predict the longshore current velocity at the breaking point, especially for mixed and gravel beaches. The new improved equation was compared with published laboratory data. Despite the fact that the new equation showed better results than the modified equation of Longuet-Higgins, this equation needs to be investigated further.

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References

Antoniadis C., 2009, Wave-induced currents and sediment transport on gravel and mixed beaches, Ph.D. Thesis, Cardiff University.

Battjes, J.A., 1974. Surf-Similarity. Proceedings of the 14th Coastal Engineering Conference, ASCE, pp.466-480.

Dean, R.G., 1965. Stream function representation of nonlinear ocean waves. *J. Geoph. Res.*, 70 (8): 4561–4572.

Galvin, C.J., 1968. Breaker Type Classification on Three Laboratory Beaches. *J. Geoph. Res.*, 73 (12): 3651-3659.

Galvin, C.J., 1969. Breaker travel and choice of design wave height. *Journal of Waterway Harbors Div.*, ASCE, 95, WW2, pp.175-200

Goda, Y., 1964. Wave forces on a vertical circular cylinder: Experiments and a proposed method of wave force computation. *Report of the Port and Harbor Research Institute*, Ministry of Transportation, No.8, pp. 74.

Goda, Y., 1970. A synthesis of breaker indices. Trans. JSCE, 2, pp.227-230.

Goda, Y., 1985. *Random Seas and Design of Maritime Structures*. University of Tokyo Press., ISBN 0-86008-369-1, Tokyo, 464p.

Isobe, M., 1985. Calculation and application of first-order cnodial wave theory. *Coastal Engineering Journal*, 9 : 309-325.

Isobe, M., 1987. A parabolic equation model for transformation of irregular waves due to refraction, diffraction and breaking. Coastal Engineering in Japan, JSCE, 30: 33-47.

Iversen, H.W., 1952. Laboratory study of breakers, Gravity Waves, Circular 52, US Bureau of Standards, pp.9-32.

Komar, P.D., and Gaughan, M.K., 1973. Airy wave theory and breaker height prediction. Proceedings of the 13th Coastal Engineering Conference, ASCE, pp.405-418.

Komar, P.D., 1976. Beach Processes and Sedimentation. Prentice-Hall, Englewood Cliffs, NJ.

Longuet-Higgins, M.S., 1970. Longshore currents generated by obliquely incident sea waves. *J. Geoph. Res.*, 75: 6778-6789

Longuet-Higgins, M.S., 1972. Recent Progress in the Study of Longshore Currents. Waves on Beaches, ed. R. E. Meyer, pp. 203-248. New York: Academic Press

Mitsuyasu, H., 1962. Experimental study on wave force against a wall. *Report of the Transportation Technical Research Institute*, No.47, p. 39 (in Japanese).

Munk, W.H., 1949. The solitary wave theory and its application to surf problems. Ann. New York Acad. Sci., 51, pp.376-423

Rattanapitikon, W., 1995. *Cross-Shore Sediment Transport and Beach Deformation Model*. Dissertation, Dep. Civil Engineering, Yokohama National University, Yokohama, Japan, 90p.

Rattanapitikon W., and Shibayama T., 2000. Verification and modification of breaker height formulas. *Coast. Eng. J.*, 42 (4): 389-406.

Rattanapitikon W., and Shibayama T., 2006. Breaking wave formulas for breaking depth and orbital to phase velocity ratio. *Coast. Eng. J.*, 48 (4): 395-416.

Shuto, N., 1974. Non-linear long waves in channel of variable section. *Coastal Engineering in Japan*, JSCE, 17: 1-12.

Stokes, G.G., 1847. On the theory of oscillatory waves. *Trans. Camb. Phil. Soc.*, 8, pp.411-455.

Torrini, L., and Allsop, N.W.H., 1999. Goda's breaking prediction method- A discussion note on how this should be applied. *HR Report, IT 473*, Wallingford, U.K.

Watanabe, A., Hara, T. & Horikawa, K., 1984. Study on breaking condition for compound wave trains. *Coastal Engineering in Japan*, JSCE 27, pp. 71-82.

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APPENDIX A

The application of Goda's breaking method in spreadsheet (Torrini and Allsop, 1999)

Goda's breaking method requires offshore wave conditions. In the event that the given wave height is not offshore, a synthetic one is produced, as explained below in the following:
The local wave height at a given water depth is given. The deepwater wavelength is calculated (Eq.A.1), and the breaker limit wave height is estimated using Goda's breaking criterion (Eq. A.2).

$$L_0 = \frac{gT_p^2}{2\pi}$$

(A.1)

where L_0= offshore wavelength
T_p= peak period

$$\frac{H_b}{L_c} = A\left\{1 - \exp\left[-1.5\frac{\pi d_b}{L_o}\left(1 + 15\left(\frac{1}{m}\right)^{\frac{4}{3}}\right)\right]\right\}$$

(A.2)

where H_b= breaking wave height
A = coefficient set equal to 0.12
m = bed slope (1:)

The given wave height is compared with the limiting wave height, and a warning is given if this has been exceeded; in this case, there is no need to proceed with the method.
If the initial wave height is smaller than the limiting wave height, the local wavelength is determined, using either Fenton's formula (Eq.A.3), for intermediate water, or the formula for shallow water (Eq.A.4).

$$L_{local} = L_0 \tanh\frac{2}{3}\left(\frac{2\pi h_{local}}{L_o}\right)^{\frac{3}{4}} \qquad 0.04 \leq \frac{h_{local}}{L_{local}} \leq 0.5$$

(A.3)

$$L_{local} = T\sqrt{gh_{local}} \qquad \frac{h_{local}}{L_{local}} < 0.04$$

(A.4)

where L_{local}= wavelength calculated at a given water depth
h_{local}= initial water depth

The shoaling coefficient K_s is then estimated. Since non-linear effects can be neglected in relative deep water (Goda, 1985), the shoaling coefficient is calculated here using the small amplitude wave theory (Eq.A.5).

$$K_s = \frac{1}{\sqrt{\left[1 + \frac{\left(\frac{4\pi h_{local}}{L_{local}}\right)}{\sinh\left(\frac{4\pi h_{local}}{L_{local}}\right)}\right]\tanh\left(\frac{2\pi h_{local}}{L_{local}}\right)}}$$

(A.5)

From the relationship relating the offshore wave height to the local wave height (Eq.A.6) a synthetic offshore wave height is derived.

$$H_{s0} = \frac{H_{slocal}}{K_s}$$

(A.6)

The equivalent significant deepwater wave height (significant deepwater wave height after being refracted) is calculated (Eq.A.7).

$$H_{s0}' = K_r H_{s0}$$

(A.7)

where K_r= refraction coefficient
Coming inshore, the shoaling coefficient (Shuto's non-linear shoaling coefficient, as suggested in Goda (1985) is then estimated (Eq.A.8) and the wave height is determined (Eq.A.9).

$$K_s = K_{si} \qquad \text{for } h_{30} \leq h$$

$$K_s = (K_{si})_{30}\left(\frac{h_{30}}{h}\right)^{\frac{2}{7}} \qquad \text{for } h_{50} \leq h \leq h_{30}$$

(A.8)

$$K_s\left(\sqrt{K_s} - B\right) - C = 0 \qquad \text{for } h < h_{50}$$

where h= water depth
K_{si} = shoaling coefficient for small amplitude wave (Eq.A.5)
h_{30} = water depth satisfying Eq. (A.9)
$(K_{si})_{30}$= shoaling coefficient for h_{30}
h_{50} = water depth satisfying Eq. (A.10)
B, C = constants defined in Eq. (A.11) and Eq. (A.12)

$$\left(\frac{h_{30}}{L_o}\right)^2 = \frac{2\pi}{30}\frac{H_{s0}'}{L_o}(K_{si})_{30}$$

(A.9)

$$\left(\frac{h_{50}}{L_o}\right)^2 = \frac{2\pi}{50}\frac{H_{s0}'}{L_o}(K_{si})_{50}$$

(A.10)

$$B = \frac{2\sqrt{3}}{\sqrt{\frac{2\pi H_{s0}'}{L_0}}}\frac{h}{L_0}$$

(A.11)

$$C = \frac{C_{50}}{\sqrt{\frac{2\pi H_{s0}'}{L_0}}}\left(\frac{L_0}{h}\right)^{\frac{3}{2}}$$

(A.12)

where $(K_{si})_{30}$= shoaling coefficient at $h=h_{50}$
C_{50} = constant defined by Eq. (A.13)

$$C_{50} = (K_{si})_{50}\left(\frac{h_{50}}{L_0}\right)^{\frac{3}{2}}\left[\sqrt{2\pi\frac{H_{s0}'}{L_0}(K_{si})_{50}} - 2\sqrt{3}\frac{h_{50}}{L_0}\right]$$

(A.13)

The wave height is then estimated by shoaling, Eq. (A.14) and compared with the breaker limit wave height, calculated using Goda's breaking criterion (Eq. A.2).

$$H_{si} = K_s H_{s0}'$$

(A.14)

When the limit is exceeded, breaking is initiated, the wave has entered the surf zone and Goda's braking method is applied (Eq. A.15).

$$H_{1/3} = \begin{cases} K_s H_{s0}' & \text{for } \frac{h}{L_0} \geq 0.2 \\ \min\{(\beta_0 H_{s0}' + \beta_1 h), \beta_{max} H_{s0}', K_s H_{s0}'\} & \text{for } \frac{h}{L_0} < 0.2 \end{cases}$$

(A.15)

where β_0, β_1, and β_{max} are defined as follow:

$$\beta_0 = 0.028\left(\frac{H_{s0}}{L_0}\right)^{-0.38}\exp(20m^{1.5})$$

(A.16)

$$\beta_1 = 0.52e^{4.2m}$$

(A.17)

$$\beta_{max} = \max\left\{0.92, 0.32\left(\frac{H_{s0}}{L_0}\right)^{-0.29}e^{2.4m}\right\}$$

(A.18)