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Heat Transfer over a Stretching Surface with Variable Thickness Embedded in Porous Medium in the Presence of Maxwell Fluid

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Abstract

The heat generation/absorption effects in a Maxwell fluid over a stretching surface with variable thickness embedded in a porous medium is considered. The nonlinear partial differential equations are transformed into nonlinear ordinary differential equations by similarity method. The resulting coupled nonlinear ordinary differential equations are solved under appropriate transformed boundary conditions using the Runge-Kutta fourth order along with shooting method. Comparisons with previously published work are performed and the results are found to be in very good agreement. Characteristics of dissonant parameters on velocity, temperature, skin friction coefficient and Nusselt number are collected and discussed through graphs and tables.

Keywords: Maxwell Fluid; Stretching surface; Porous medium; Heat generation/absorption

Nomenclature: a,b=Constants, A=Very small constant, C_p=Specific heat at constant pressure, J/Kgk, C_f= Local skin-friction coefficient, f, F=Dimensionless functions, k=Thermal conductivity, k_∞=Ambient thermal conductivity, K= Permeability of porous medium, m^2, k_0 = Reference permeability of porous medium, n=Velocity power index, Nu= Nusellt number, Pr= Prandtle number, Q= Rate of Heat generation/absorption, Re_a=Local Reynold's number, T=The temperature, K, T_∞=Ambient temperature, U_w=Velocity of solid surface, u, v=Velocity components along x and y, m/s, x, y=Distance along and normal to the surface. Greek Symbols: α=Wall thickness parameter, β=The elasticity parameter, γ=Heat generation/absorption parameter, ε=Thermal conductivity parameter, n=Dimensionless coordinate, θ, φ=Dimensionless temperature, λ=Porous parameter, λ_1=The relaxation time of the fluid, v=Kinematical Viscosity=u/ρ, m/s^2, ρ=Fluid density, kg/m^3, u=Dynamical viscosity, kg/ms, ϕ=Physical stream function. Subscripts: w= Condition on the wall, ∞= Free stream (ambient) condition, o= Reference value.

Introduction

Stretching surface in a stationary cooling fluid is a main stage in a lot of manufacturing processes such as plastic sheets, glass manufacturing, etc. The simplicity of geometry of such a problem encourages a lot of researchers to study the flow and heat transfer of the boundary layer generated above the stretched surface. The Newtonian fluid above a flat surface is considered by Crane [1] who presented a simple closed exponential form for the two-dimensional flow of Newtonian fluid treating the simplest heat conduction equation due to linear, then Chiam [2] studied the effects due to the dissipation, stress work and heat generation. Elbashbeshy and Bazid [3] considered the porous medium effect to present a new similarity solution for the temperature filed for unsteady flow, which transforms the time dependent thermal energy equation into an ordinary differential equation. The porous medium effect for the Newtonian fluid also has been studied by Cortell [4] and Yousof [5]. Elbashbeshy [6] considered the thermal radiation, heat generation, and the free convection effects within the steady flow over exponentially stretching surface. Swati and Mandal [7] introduced again the work of Elbashbeshy [6] in porous medium. Eid [8] studied the slip velocity effect. The existence of heat generation/ absorption source has been considered by Chiam [2], Cortell [4] and Elbashbeshy [6].

The flow due to stretching flat surface in non-Newtonian fluid takes place in a lot of extrusion process. The Maxwell fluid in a porous medium for steady flow has been considered by Hossain [9], Swati et al. [10], Shehzad et al. [11], Singh and Agarwal [12]. Waheed [13] studied the unsteady flow. Noor [14] studied heat generation source.

The engineering applications in stretching surface with variable thickness exist more frequently than flat surfaces. The non-Flatness effects on the flow of Newtonian fluid above stretching surface with variable thickness was considered by Fang [15]. Furthermore, khader [16-18], Elbashbeshy [19] and Sanadeep [20]. Wahed [21] studied the nanofluids in the presence of Brownian motion for stretching surface of variable thickness. khader [22] considered the MHD fluid above stretching surface of variable thickness.

There is no investigation for non-Newtonian fluid over stretching surface with variable thickness except by Hayat [23] for Cattaneo-Christov heat flux. In this paper we will considered the flow and heat transfer of a Maxwell fluid over as stretching surface with variable thickness embedded in a porous medium, also the existence of heat generation source will be considered.

The main objective is to introduce a numerical analysis for the porous medium effect and the heat source effect on the flow and heat transfer. The schematic of the process is shown in Figure 1, where the melted material is extruded from a die and then it is stretched by wind-up roll. An induced motion in the above stationary fluid is generated. The thickness of the stretched surface is assumed to decrease gradually until get a uniform thickness (Figure 1). The numerical results will be presented in tables in comparison with others and will be shown in graphs.

Mathematical Formulation

Consider a steady, two-dimensional laminar boundary layer flow of

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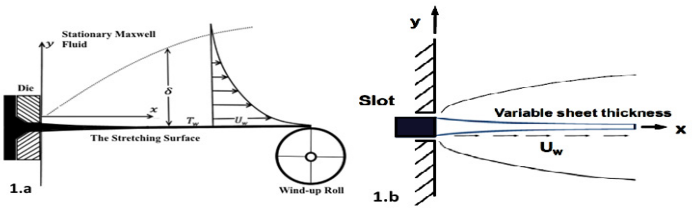

Figure 1: Schematic for flow above stretching surface.

Maxwell fluid over a stretching surface in porous medium. The surface profile is described with $y=A(x+b)^{\frac{1-n}{2}}$ and the stretching velocity is $U_w=a(x+b)^n$, where $a\ and\ b$ are constants, n is the velocity power index, and A is very small constant. The surface is assumed impermeable $v_w = 0$. The x-axis is chosen along the stretching surface in the direction of the motion and y-axis is taken normal to it. The equations governing the flow in the boundary layer of a steady, laminar and incompressible Maxwell fluid are

$$\frac{\partial u}{\partial u}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left[u^2\left(\frac{\partial^2 u}{\partial x^2}\right)+v^2\left(\frac{\partial^2 u}{\partial y^2}\right)+2uv\left(\frac{\partial^2 u}{\partial x\partial y}\right)\right]=v\frac{\partial^2 u}{\partial y^2}-\frac{v}{k}u \quad (2)$$

$$\begin{aligned}&at y=A(x+b)^{\frac{\cdot\cdot}{2}}:u=U_w=a(x+b)^n, v=0, T=T_w\\&at\ y\to\infty:u\to o, T\to T\infty\end{aligned} \quad (3)$$

With boundary conditions

$$\begin{aligned}&at y=A(x+b)^{\frac{1-n}{2}}:u=U_w=a(x+b)^n, v=0, T=T_w\\&at\ y\to\infty:u\to o, T\to T\infty\end{aligned} \quad (4)$$

where u and v are the velocity components along x and y respectively, t is the time, λ_1 is the relaxation time parameter and T is the temperature. The heat generation is $(x)=Q_0(x+b)^{n-1}$ [9]. The fluid properties are p as the density, v as the dynamic viscosity, k as the thermal conductivity, $_{\text{Cp as}}$ the specific heat at constant pressure and $K=K_0(x+b)^{1-n}$ [20] is the permeability of the porous medium. T_w is the wall surface temperature, and T is ambient temperature.

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables [9,23]

$$n=\sqrt{\left(\frac{n+1}{2}\right)\frac{a(x+b)^{n-1}}{v}}\,y;\psi=\sqrt{\frac{2}{n+1}va(x+b)^{n+1}}.F(n)=\frac{T-T_w}{T_w-T_\infty} \quad (5)$$

Where F (n) and φ (n) are dimensionless functions. The equations 2 and 3 were transformed into ordinary differential equations using the similarity transformation technique. The equation of continuity is satisfied if we choose a stream function ϕ such that

$$u=\frac{\partial\psi}{\partial y}=a(x+b)^n F', v=-\frac{\partial\psi}{\partial x}=-\sqrt{\frac{n+1}{2}va(x+b)^{n-1}}\left(F+\frac{n-1}{n+1}nF'\right) \quad (6)$$

Guaranteed that u and in v equation 6 satisfied the continuity equation, see equation 1. The thermal conductivity is generalized as follow:

$$k=k_\infty(1+\varepsilon\phi) \quad (7)$$

Where ε is the thermal conductivity parameter and k_∞ is ambient thermal conductivity?

Substituting (5), (6) and (7) in (2) and (3) we obtain the following ordinary differential equations:

$$\begin{aligned}&F'''+FF''-\frac{2n}{n+1}F'^2+\\&\beta\left((3n-1)FF'F''-\frac{2n(n-1)}{n+1}F'^3-\frac{n-1}{2}nF^2F'''-\frac{n+1}{2}F^2F'''\right)\end{aligned} \quad (8)$$

$$\left(\frac{1}{\Pr}\right)\left[(1+\varepsilon\phi)\phi''+\varepsilon\phi'^2\right]+F\phi'+\gamma\phi=0 \quad (9)$$

with boundary conditions

$$\begin{aligned}&F(\alpha)=\alpha\left(\frac{1-n}{1+n}\right), F'(\alpha)=1, \phi(\alpha)=1\\&F'(\infty)=0, \phi(\infty)=0\end{aligned} \quad (10)$$

where the wall thickness parameter $\alpha=A\sqrt{a(n+1)/2v}$ describes the wall profile of the stretched surface, is the elasticity number [23], $\lambda=2v/ak_0(n+1)$ is the porous parameter, $\Pr=uC_p/k_\infty$ is the Prandtle number, $\gamma=2Q_0/\rho C_p a(n+1)$ is the heat generation $(\gamma<0)$ and absorption $(\gamma<0)$ parameter.

If we use the following transformation $F(n-\alpha)=f\ and\ \phi(n-\alpha)=\theta$, then the equations (8) and (9) becomes

$$\begin{aligned}&f'''+ff''-\frac{2n}{n+1}f'^2+\\&\beta\left((3n-1)fff''-\frac{2n(n-1)}{n+1}f'^3-\frac{n-1}{2}nf^2f'''-\frac{n+1}{2}f^2f'''\right)\end{aligned} \quad (11)$$

$$\left(\frac{1}{\Pr}\right)\left[(1+\varepsilon\theta)\theta''+\varepsilon\theta'^2\right]+f\theta'+\gamma\theta=0 \quad (12)$$

With boundary conditions

$$\begin{aligned}&f(0)=\alpha\left(\frac{1-n}{1+n}\right), f'(0)=1, \theta(0)=1\\&f'(\infty)=0, \phi(\infty)=0\end{aligned} \quad (13)$$

The following physical quantities takes place in our study

$$C_f = -2\sqrt{\frac{n+1}{2}}\,\mathrm{Re}_x^{-\frac{1}{2}} f''(0),\; Nu = -\sqrt{\frac{n+1}{2}}\,\mathrm{Re}_x^{-\frac{1}{2}}\theta'(0) \quad (14)$$

We will present the $C_f\sqrt{\mathrm{Re}_x} = 2\sqrt{(n+1)}/2f''(0)$ as the reduced skin-friction, and $Nu\sqrt{\mathrm{Re}_x} = -\sqrt{(n+1)}/2\theta'(0)$ as the reduced Nusselt number. Where C_f is the local skin-friction coefficient, N_u is the local Nusselt number and $\mathrm{Re}_x = U_w(x+b)/v$ is the local Reynold number [16].

Results and Discussion

In this section, we analyzed and discuss our numerical results. At first, a comparison with others for $-f''(0)$ at different values of α and for $-\theta'(0)$ when n=1 (flat surface). Second, Graphs show the influence of elasticity β wall thickness α porous λ parameters, and the velocity power index n on the velocity and temperature profiles. Other graphs display the influence of thermal conductivity ε, heat generation/absorption γ parameters and Pr Prandtle number on the temperature profile. Finally, tables present the induced skin-friction and induced Nusselt number.

Table 1 disclose that our numerical results for $-f''(0)$ at different α (neglecting the effect of the porous medium) shows an excellent agreement with Fang [17] and Khader et al., [18] for four digits at different values of *n*. Table 2 displays $-\theta'(0)$ for flat surface $n=1, \gamma=0$ in comparison with Salahuddin et al. [24], Mabood et al. [25] and Wang [26]. The table also shows an excellent agreement with them. So, Tables 1 and 2 guarantee the results of the Mathematica software used in this paper.

The elasticity parameter β is also known as the Maxwell relaxation parameter. This parameter measures the deviation from the Newtonian behavior to the non-Newtonian for the fluid's flow. Figure 2 shows the influence of β on the velocity profile. From Figure 2, it is noticed that increasing the relaxation time β cause a resistance reduces the velocity and hence the boundary layer thickness, it noted also that the porous medium encourages β effect in lowering the velocity. From Figure 3, in the existence of heat absorption source, it is noticed that the effect of increasing values of the relaxation time β increases the temperature distribution within the fluid while the existence of heat generation source reduces it.

The effect of wall thickness parameter α on the fluid flow and the temperature profiles have been investigated and the results are presented in Figures 4-7. From Figure 4, it is clear that for n<1, as α increases the velocity slightly decreases, consequently, the boundary layer becomes thinner and thinner, while Figure 6 shows that for n>1 the velocity increases and the boundary layer thickness become thicker.

On the other hand, Figure 5 presents the influence of α on the temperature profile for n<1, as α increases the temperature decreases due to the slowdown in the flow. While Figure 7 show the increase in the temperature with increasing α for n>1. From Figures 4-7, it is noticed that the porous medium increases the α effect.

Effects of the porous parameter λ on velocity and temperature profiles are shown in Figures 8 and 9, respectively. From Figure 8, the horizontal velocity decreases for increasing values of porous parameter due to the increase of the pores in the stretched surface. Furthermore, the momentum boundary layer thickness decreases as porous parameter λ, it is also noticed the λ effect for Newtonian fluid is lower than for Maxwell fluid. Figure 9 shows the increase in the temperature due to the increase in the porous parameter in the existence of heat generation/absorption source.

The velocity index power n describes the surface profile and the stretching wall velocity. From Figure 10, it is clear that, the velocity is slightly decreases with increasing the velocity power index n, and this effect increases in the porous medium. Figure 11 shows the influence *n* on the temperature profiles, it is clearly seen that increasing the value of *n* produces an increase in the temperature profiles. It further shows that the larger of the value of n,the thermal boundary layer thickness increases. It is also noticed that the existence of heat generation source encourages this influence of n than the heat absorption source.

The thermal conductivity parameter ε describes the thermal conductivity in the fluid. Figure 12(a) shows an increase in the temperature with increasing the ε parameter and the porous medium

n	α= 0.25			α= 0.5		
	T.Fang [17]	Khader et al.[18]	Present Work	T.Fang [17]	Khader et al. [18]	Present Work
10	1.1433	1.1433	1.14332	1.0603	1.0603	1.06032
9	1.1404	1.1404	1.14039	1.0589	1.0588	1.05891
7	1.1323	1.1322	1.13228	1.0550	1.0551	1.05504
5	1.1186	1.1186	1.11859	1.0486	1.0486	1.04861
3	1.0905	1.0904	1.09049	1.0359	1.0358	1.03586
1	1.0000	1.0000	1.00000	1.0000	1.0000	1.00000
0.5	0.9338	0.9337	0.93382	0.9799	0.9798	0.9798
0	0.78439	0.7843	0.78427	0.9576	0.9577	0.95764
-1/3	0.5000	0.5000	0.50000	1.0000	1.0000	1.00000
-0.5	0.0833	0.0832	0.08333	1.1667	1.1666	1.16666

Table 1: A comparison with others for the value of $-f''(0)$ at different α.

Pr	Salahuddin et al. [24]	Mabood et al. [25]	Wang [26]	Present Work
0.07	0.0654	0.0655	0.0656	0.0656225
0.2	0.1688	0.1691	0.1691	0.1690885
0.7	0.4534	0.4539	0.4539	0.4539165
2	0.9108	0.9114	0.9114	0.9113577
7	1.8944	1.8954	1.8954	1.8954033
20	3.3522	3.3539	3.3539	3.3539043
70	6.4619	6.4622	6.4622	6.4621999

Table 2: Comparison of $-\theta'(0)$ when α = β = γ = ε = λ = 0 and n = 1.

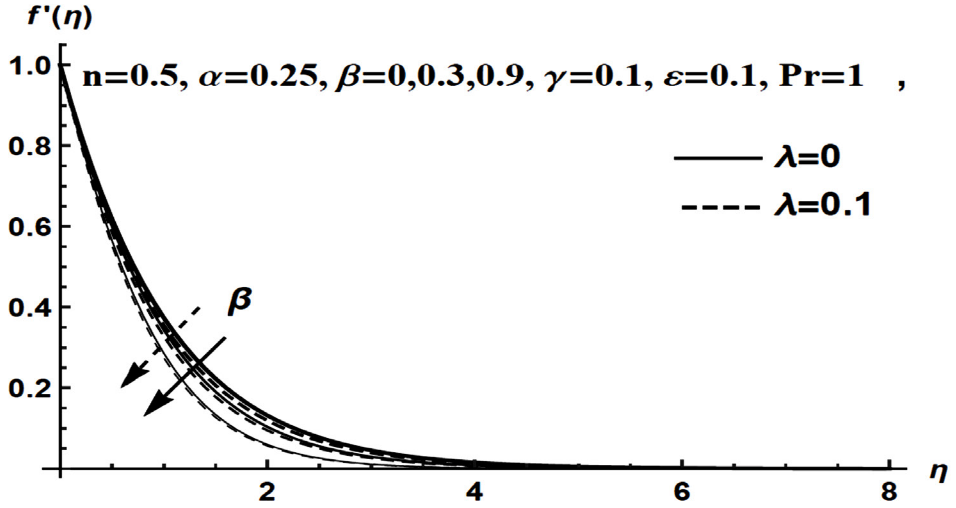

Figure 2: The elasticity number effect on the velocity with and without porous medium.

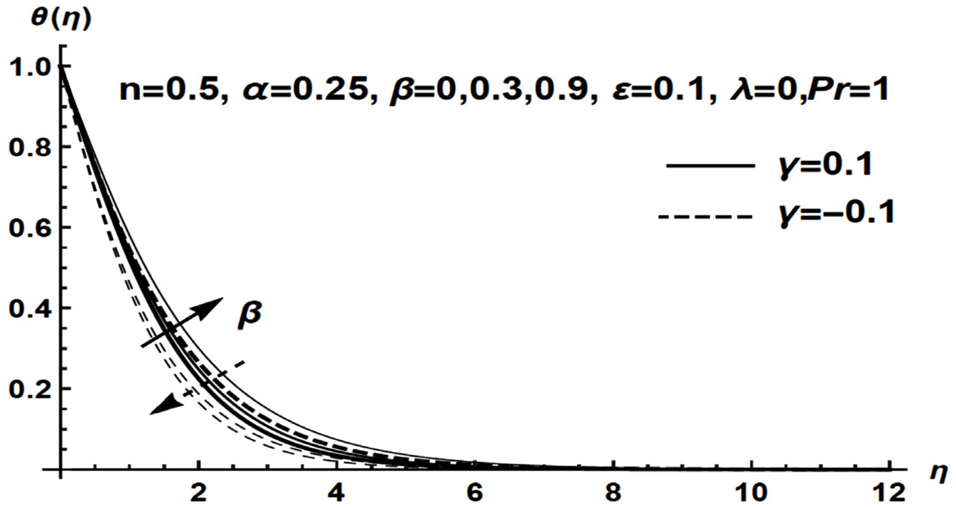

Figure 3: The elasticity number parameter effect on the temperature with generation/absorption.

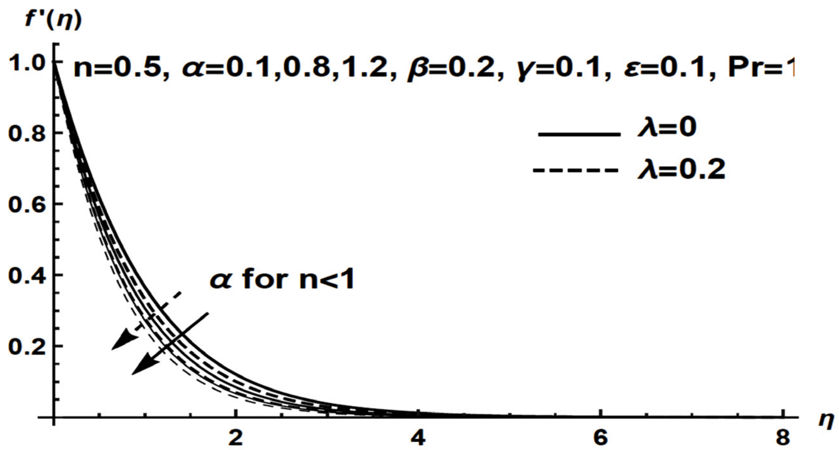

Figure 4: The wall thickness parameter effect on the velocity with/without porous medium.

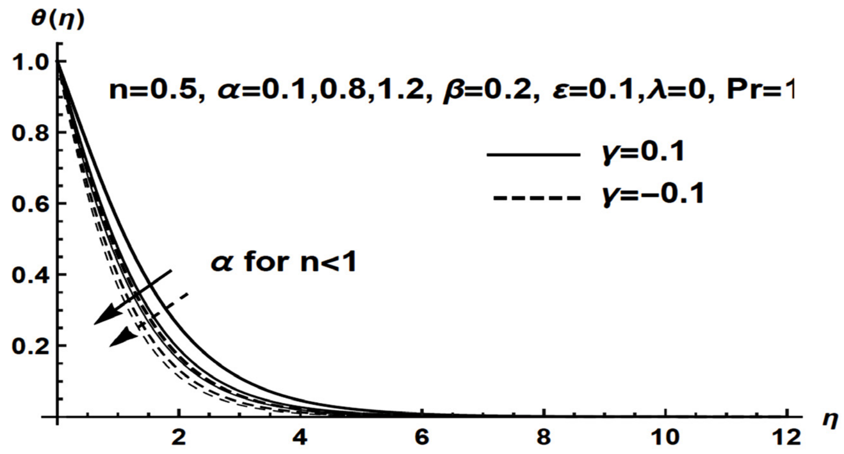

Figure 5: The wall thickness parameter effect on the temperature with generation/absorption.

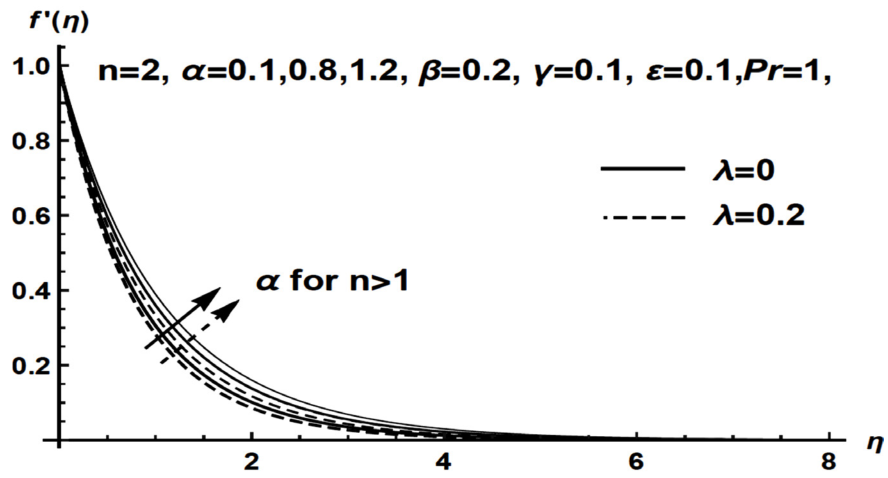

Figure 6: The wall thickness parameter effect on the velocity considering the porous medium effect.

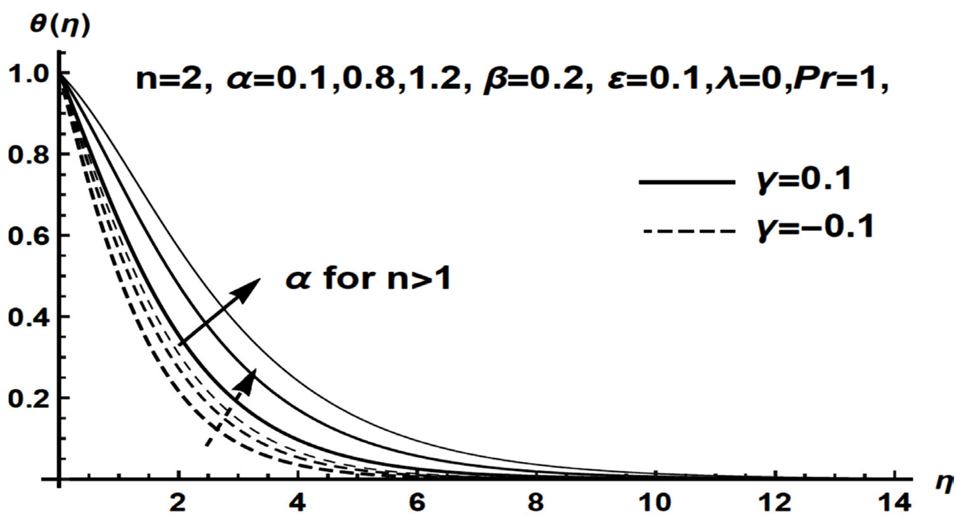

Figure 7: The wall thickness parameter effect on the temperature considering the porous medium effect.

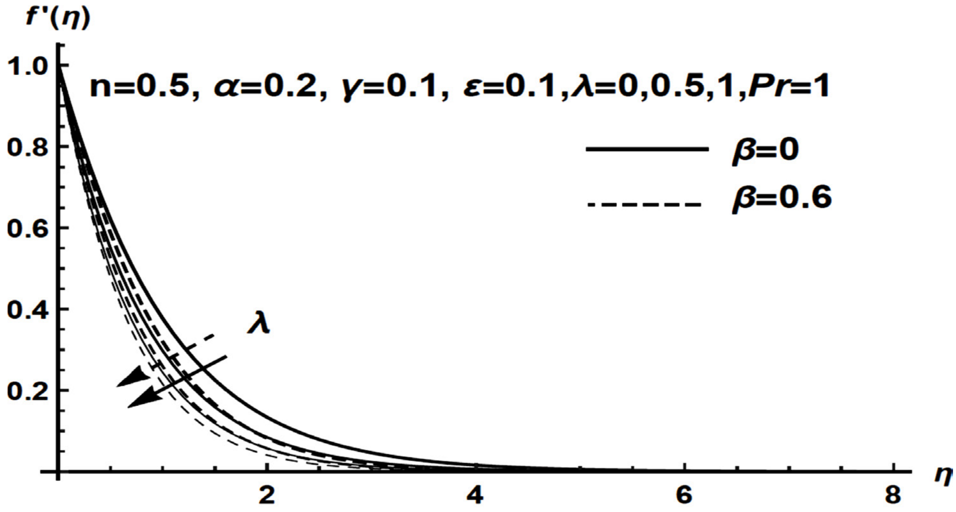

Figure 8: The porous parameter effect on the velocity considering the elasticity parameter.

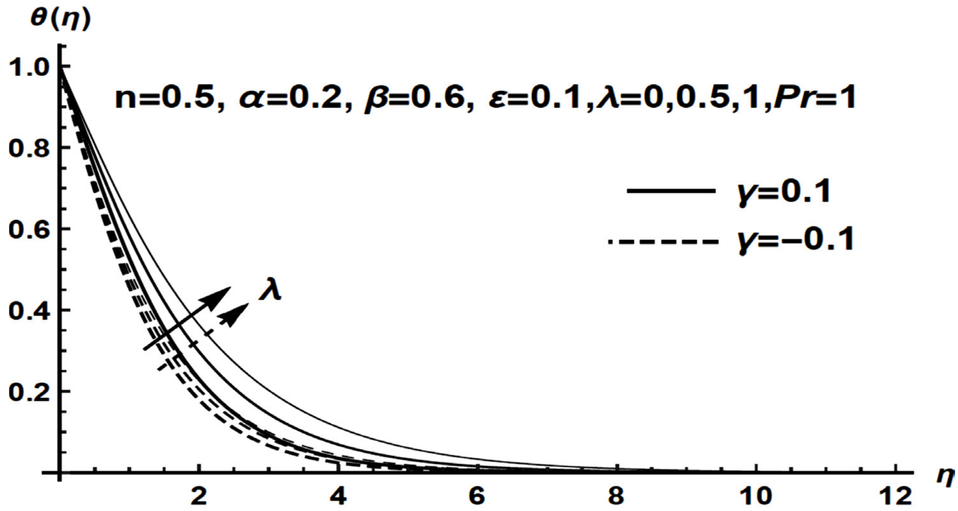

Figure 9: The porous parameter effect on the temperature considering the elasticity parameter.

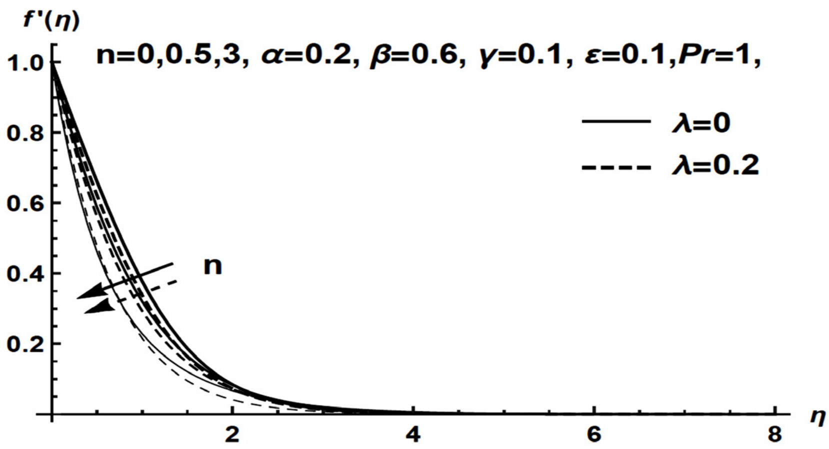

Figure 10: The velocity power index effect on the velocity.

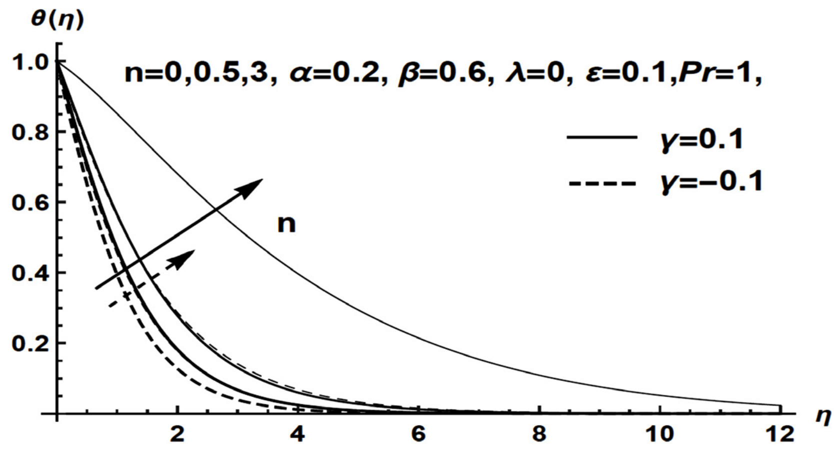

Figure 11: The velocity power index effect on the temperature.

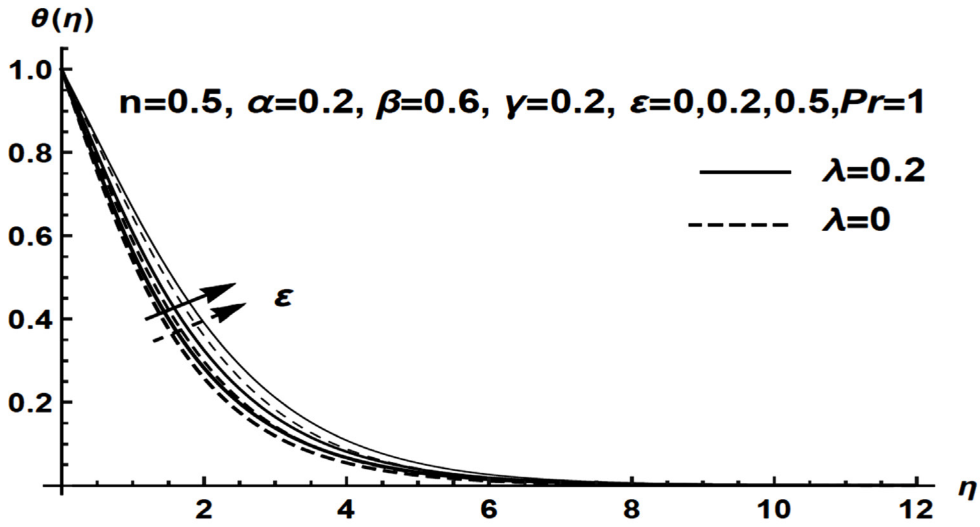

Figure 12a: The effect of thermal conductivity parameter on the temperature.

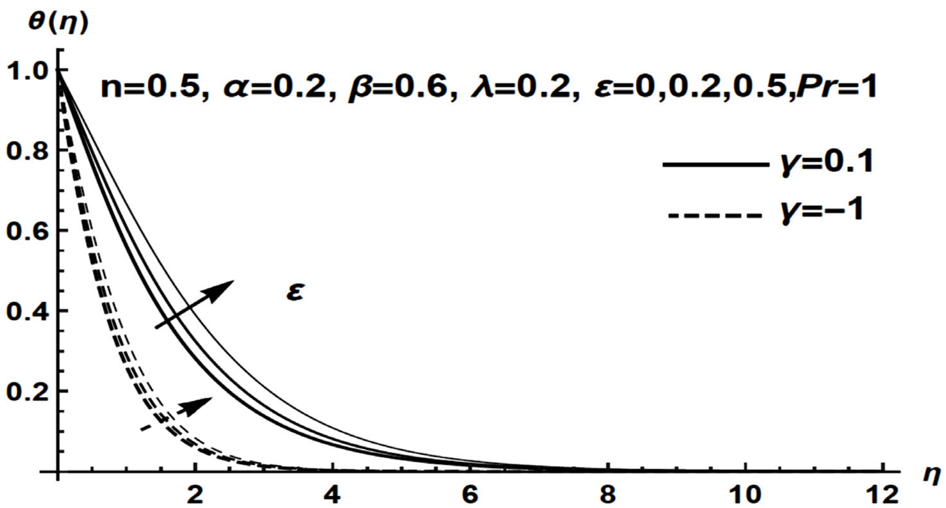

Figure 12b: The effect of thermal conductivity parameter on the temperature.

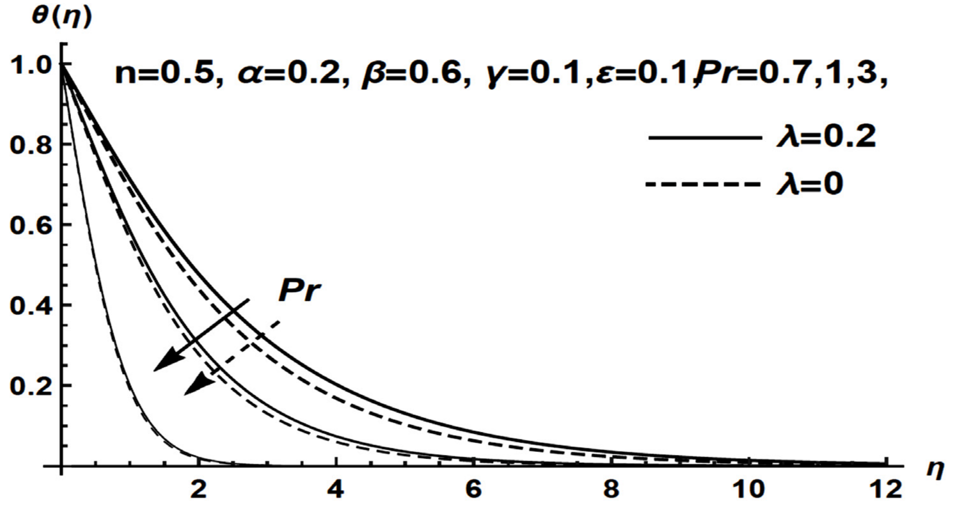

Figure 13a: The effect of Prandtle number on the temperature.

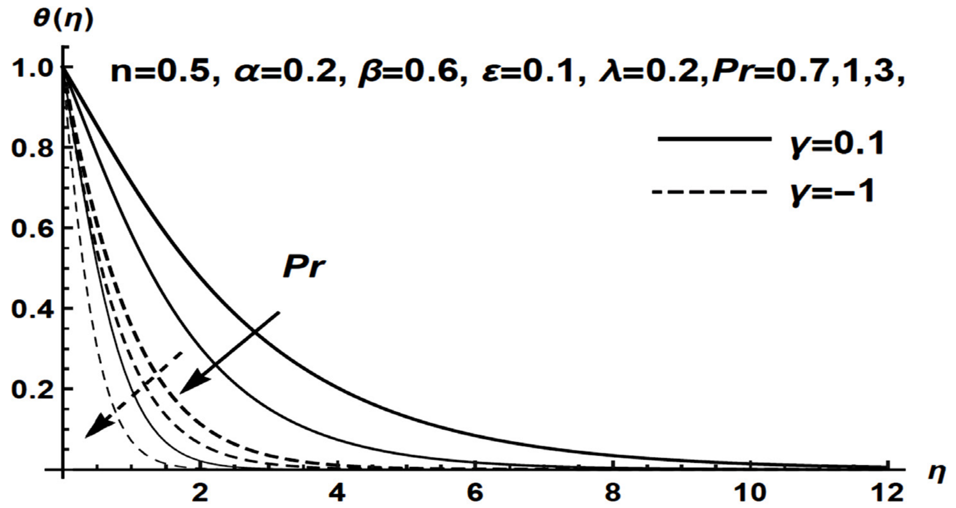

Figure 13b: The effect of Prandtle number on the temperature.

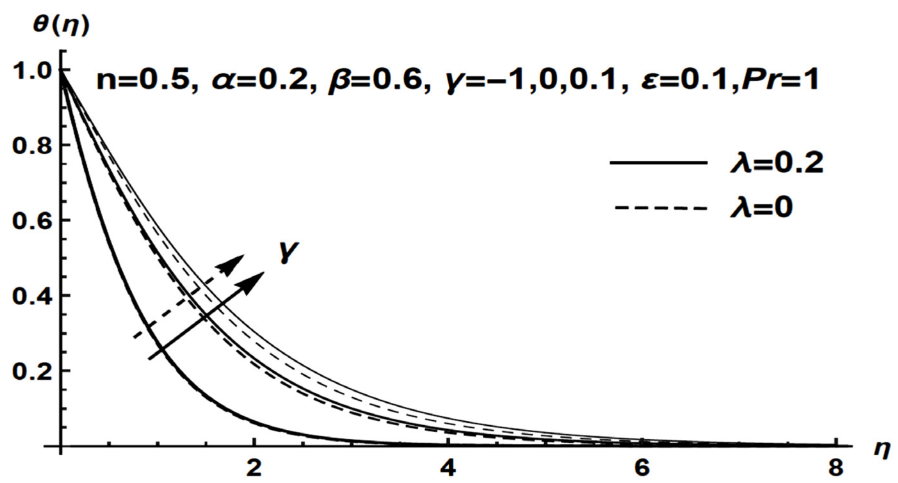

Figure 14: The effect of the heat generation parameter on the temperature.

N	α	β	λ	ε	Pr	γ	$C_f\sqrt{Re_x}$	$Nu\sqrt{Re_x}$
	0.2	0.3	0.2	0.1	1	0.1	0.854898	0.583417
0.5	0.2	0.3	0.2	0.1	1	0.1	1.048258	0.451196
5	0.2	0.3	0.2	0.1	1	0.1	1.623654	0.089902
0.5	0	0.3	0.2	0.1	1	0.1	1.002188	0.410101
0.5	0.25	0.3	0.2	0.1	1	0.1	1.060248	0.418234
5	0	0.3	0.2	0.1	1	0.1	1.870432	0.148667
5	0.25	0.3	0.2	0.1	1	0.1	1.569894	0.053336
0.5	0.2	0	0.2	0.1	1	0.1	1.029135	0.470600
0.5	0.2	0.9	0.2	0.1	1	0.1	1.090630	0.408028
0.5	0.2	1.4	0.2	0.1	1	0.1	1.129228	0.367230
0.5	0.2	0.3	0	0.1	1	0.1	0.947024	0.476771
0.5	0.2	0.3	0.5	0.1	1	0.1	1.185025	0.414967
0.5	0.2	0.3	1	0.1	1	0.1	1.384019	0.357501
0.5	0.2	0.3	0.2	0	1	0.1	1.048258	0.490482
0.5	0.2	0.3	0.2	0.2	1	0.1	1.048258	0.417748
0.5	0.2	0.3	0.2	0.5	1	0.1	1.048258	0.341185
0.5	0.2	0.3	0.2	0.1	0.7	0.1	1.048258	0.311545
0.5	0.2	0.3	0.2	0.1	3	0.1	1.048258	1.067937
0.5	0.2	0.3	0.2	0.1	7	0.1	1.048258	1.881337
0.5	0.2	0.3	0.2	0.1	1	-1	1.048258	1.130866
0.5	0.2	0.3	0.2	0.1	1	0	1.048258	0.560210
0.5	0.2	0.3	0.2	0.1	1	0.1	1.048258	0.457829

Table 3: The reduced skin-friction and reduced Nusselt number at different values of n, α, β, λ, ε, Pr and R.

encourage that. From Figure 12(b), the heat generation/absorption source is declared. It seen that the effect of the thermal conductivity in existence of heat generation source is higher than for existence of heat absorption source.

Prandtle number Pr introduces the ratio between the momentum and the thermal diffusivities. Figures 13(a) and 13(b) show the influence of Pr on the temperature profile, such that increasing Pr decreases the temperature distribution. Physically, the increase in Pr means a decrease in the thermal diffusivity, then the fluid has larger heat capacity. The Prandtle number effect is reinforced in the porous medium and heat generation source. The heat generation/absorption parameter γ effect is shown in Figure 14. It is noticed that the increase in γ produces increase the temperature distribution, and the porous medium is slightly affect this phenomenon.

The skin-friction is a generated force in the boundary layer while flow above the stretching surfaces. Measuring the skin-friction introduces the required drag force to pull the stretched sheet and then to control the required mechanical properties. Table 3 presented the values for the reduced skin friction at different settings. It is shown that the skin-friction increases and the reduced Nusselt number decreases with increasing the velocity power index n, elasticity number β, and porous parameter λ. The wall sickness parameter α has vice versa effect for n>1 against n<1on the reduced skin-friction and Nusselt number and the Prandtle number. The thermal conductivity parameter ε and the Prandtle number Pr increases the reduced Nusselt number, while the thermal radiation parameter decreases it, but the previous parameters have no effect on the skin-friction.

Conclusions

In this paper, the Maxwell fluid over stretching surface with variable thickness is solved numerically. The effects of the variable wall thickness, the heat generation/absorption source, and the porous medium on the flow and heat transfer at different settings are investigated numerically. The results in comparison with previous results presented an excellent agreement. From the previous results that mentioned parameters in tables and graphs. We concluded that:

- The elasticity parameter β, decreases the velocity profile and the temperature profile. The effect of the elasticity parameter on the temperature for the heat generation is inversed for the heat absorption existence.
- The wall thickness parameter α, (for $n > 1$) slightly decreases the velocity and the temperature, while when it strongly increases them. The porous medium reinforces the wall thickness parameter effect in lowering the velocity for and increasing the velocity for $n > 1$. It is also seen that the heat generation effect on the influence of α for n>1 is greater than for n<1.
- The porous parameter λ, decreases the velocity, but it increases the temperature, and the λ effect for Newtonian fluid is lower than for Maxwell fluid.
- The velocity power index n , decreases the horizontal velocity, which in turn reduces the boundary layer thickness, but it increases the temperature distribution. The existence of heat generation and porous medium encourage the effect on the temperature and velocity respectively.
- Increasing the thermal conductivity parameter ε increases the temperature distribution, while increasing Prandtle number Pr decreases it, which in turn reduces the thermal boundary thickness.
- The existence of heat generation source increases the temperature distribution.
- The heat transfer is enhanced by increasing the elasticity, the porous, the velocity power index, the thermal conductivity, and the heat source parameters.

- The reduced skin-friction $C_f\sqrt{\mathrm{Re}_x}$ is enhanced with increasing $n, \alpha\left(for\, n<1\right), \beta\, and\, \lambda$. While it's decreased with increasing $\alpha\left(for\, n>1\right)$.
- The reduced Nusellt number $Nu\sqrt{\mathrm{Re}_x}$ is increased by increasing $\varepsilon, \alpha\left(forn<1\right)$, whereas increasing $n, \alpha\left(for\, n>1\right),, \beta, \lambda$ and γ reduces this number.

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