

Fundamental Solution Based Finite Element Method

Qing H Qin*

Research School of Engineering, Australian National University, Canberra, Australia

A new type of hybrid finite element method with fundamental solutions as internal interpolation functions, named as HFS-FEM [1], was recently developed based on the framework of Hybrid Trefftz finite element method (HT-FEM) [2-4] and the idea of the Method of Fundamental Solution (MFS) [5,6]. In this method, two independent assumed fields (intra-element field and auxiliary frame field) are employed and the domain integrals in the variational functional can be directly converted to boundary integrals without any appreciable increase in computational effort as in HT-FEM. The assumed intra-element field was constructed by the linear combination of fundamental solutions at points outside the elemental domain under consideration. The independent frame field was introduced to guarantee the inter-element continuity and the stationary condition of the new variational functional is used to obtain the final stiffness equations.

Historically, research into the development of efficient finite elements has mostly concentrated on the following three distinct types of FEM [2,7-10]. The first is the conventional FEM. It is based on a suitable polynomial interpolation which has already been used to analyze most engineering problems. With this method, the solution domain is divided into a number of small cells or elements, and material properties are defined at element level [7]. The second is the natural-mode FEM. In contrast, the natural FEM, initiated by Argyris and his co-workers [8], presents a significant alternative to conventional FEM with ramifications on all aspects of structural analysis. It makes distinction between the constitutive and geometric parts of the element tangent stiffness, which could lead effortlessly to the non-linear effects associated with large displacements. The final is the so-called hybrid Trefftz FEM (HT-FEM) [2,11]. Unlike in the conventional and natural FEM, the HT-FEM couples the advantages of FEM [7] and BEM [12]. In contrast to conventional FEM and BEM, the HT-FEM is based on a hybrid method which includes the use of an independent auxiliary inter-element frame field defined on each element boundary and an independent internal field chosen so as to a priori satisfy the homogeneous governing differential equations by means of a suitable truncated T-complete function set of homogeneous solutions. Inter-element continuity is enforced by using a modified variational principle, which is used to construct the standard force-displacement relationship and establish linkage of frame field and internal fields of the element. The property of non-singular element boundary integral appeared in HT-FEM enables us to construct arbitrary shaped element conveniently. However, the terms of truncated T-complete functions should be carefully selected in achieving desired results. Further, the T-functions may be difficult to generate for some physical problems. To remove the drawback of HT-FEM, the HFS-FEM, was recently developed for solving various engineering problems [13-19]. The proposed HFS-FEM can be viewed as the fourth type of FEM which is significantly different from the previous three types discussed above. In the analysis, a linear combination of the fundamental solution at different points is used to approximate the field variable within the element. The HFS-FEM inherits all advantages of HT-FEM and removes the difficulty occurred in constructing and selecting T-functions, so it may have more extensive applications than the HT-FEM. The employment of two independent fields also makes the HFS-FEM easier to generate

arbitrary polygonal or even curve-sided elements. It also obviates the difficulties that occur in HT-FEM [2,3] in deriving T-complete functions for certain complex or new physical problems [14]. The HFS-FEM has simpler expressions of interpolation functions for intra-element fields (fundamental solutions) and avoids the coordinate transformation procedure required in the HT-FEM to keep the matrix inversion stable [2]. Moreover, this approach also has the potential to achieve high accuracy using coarse meshes of high-degree elements, to enhance insensitivity to mesh distortion, to give great liberty in element shape, and accurately representing various local effects (such as hole, crack and inclusions) without troublesome mesh adjustment [2,19]. On the other hand, it should be pointed out that the developed HFS-FEM approach is different from the BEM [12], although the same fundamental solution is employed. Using the reciprocal theorem, the BEM obtains the boundary integral equation, which usually encounters difficulty in dealing with singular or hyper-singular integrals in practical analysis, while the weakness can be removed using HFS-FEM. Additionally, HFS-FEM makes it possible for a more flexible element material definition which is important in dealing with multi-material problems, rather than the material definition being the same in the entire domain in BEM.

As indicated in [1], The HFS-FEM of a heat conduction problem is constructed based on the modified variational below

$$\Pi_m = \sum_e \Pi_{me} = \sum_e \left\{ \Pi_e + \int_{\Gamma_e} (\tilde{u} - u) q d\Gamma \right\}$$

where

$$\Pi_e = -\frac{1}{2} \int_{\Omega_e} k u_{,i} u_{,i} d\Omega - \int_{\Gamma_{qe}} \bar{q} \tilde{u} d\Gamma$$

in which u is temperature, k is the thermal conductivity, an q represents the boundary heat flux. The boundary Γ_e of a particular element e consists of the following parts

$$\Gamma_e = \Gamma_{ue} \cup \Gamma_{qe} \cup \Gamma_{le}$$

Where Γ_{le} represents the inter-element boundary of the element 'e'.

The HFS-FEM discussed here has the following features:

- Compared to the conventional FEM, the formulation calls for integration along the element boundaries only, which simplify the calculation of stiffness matrix and is easy to generate arbitrary shaped elements.

*Corresponding author: Qing H. Qin, Research School of Engineering, Australian National University, Canberra, Australia, E-mail: qinghua.qin@anu.edu.au

Received October 14, 2013; Accepted October 14, 2013; Published October 20, 2013

Citation: Qin QH (2013) Fundamental Solution Based Finite Element Method. J Appl Mech Eng 2: e118. doi:10.4172/2168-9873.1000e118

Copyright: © 2013 Qin QH. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

- The proposed model is insensitive to the mesh distortion and can provide a good numerical accuracy.

- In contrast to the T-complete function used in HT-FEM, the fundamental solution in HFS-FEM is easy to derive, and further, the determination of source points is easier to operate than selecting appropriate terms from T-complete series in HT-FEM.

- HFS-FEM can define the fundamental solution at element level and thus can be flexibly used to analyze problems with different material properties. In contrast, BEM usually use the fundamental solutions defined in the full domain which is not convenient for such problems with different materials. Moreover, the nonsingular boundary integrals are used in the HFS-FEM, instead of singular or hyper-singular ones in the formulation of the conventional BEM.

References

1. Wang H, Qin QH (2009) Hybrid FEM with fundamental solutions as trial functions for heat conduction simulation. *Acta Mechanica Solida Sinica* 22: 487-498.
2. Qin QH (2000) *The Trefftz finite and boundary element method*. WIT Press, Southampton, UK.
3. Qin Q H, Wang H (2008) *Matlab and C programming for Trefftz finite element methods*. CRC Press, Boca Raton, USA.
4. Qin QH (2005) Trefftz finite element method and its applications. *Appl Mech Rev* 58: 316-337.
5. Wang H, Qin QH (2008) Meshless approach for thermo-mechanical analysis of functionally graded materials. *Eng Anal Bound Elem* 32: 704-712.
6. Wang H, Qin QH, Kang Y (2005) A new meshless method for steady-state heat conduction problems in anisotropic and inhomogeneous media. *Arch Appl Mech* 74: 563-579.
7. Martin HC, Carey GF (1973) *Introduction to Finite Element Analysis: Theory and Applications*. McGraw-Hill Book Company, New York, USA.
8. Argyris J H, Kleiber M (1977) Incremental formulation in nonlinear mechanics and large strain elasto-plasticity - natural approach. *Computer Methods in Appl Mech Eng* 11: 215-247.
9. Ding K, Qin QH, Cardew-Hall M (2007) Substepping algorithms with stress correction for the simulation of sheet metal forming process. *Int J Mech Sci* 49: 1289-1308.
10. Qin QH, Mao CX (1996) Coupled torsional-flexural vibration of shaft systems in mechanical engineering--I. Finite element model. *Computers & Structures* 58: 835-843.
11. Wang H, Qin QH, Arounsavat D (2007) Application of hybrid Trefftz finite element method to non-linear problems of minimal surface. *Int J Numer Meth Eng* 69: 1262-1277.
12. Qin QH (1993) Nonlinear analysis of Reissner plates on an elastic foundation by the BEM. *Int J Solids Struct* 30: 3101-3111.
13. Cao C, Qin QH, Yu A (2012) A new hybrid finite element approach for three-dimensional elastic problems. *Arch Mech* 64: 261-292.
14. Cao C, Qin QH, Yu A (2012) Hybrid fundamental-solution-based FEM for piezoelectric materials. *Comput Mech* 50: 397-412.
15. Wang H, Qin QH (2010) Fundamental-solution-based finite element model for plane orthotropic elastic bodies. *Eur J Mech A-Solid* 29: 801-809.
16. Wang H, Qin QH (2010) FE approach with Green's function as internal trial function for simulating bioheat transfer in the human eye. *Arch Mech* 62: 493-510.
17. Wang H, Qin QH, Liang X P (2012) Solving the nonlinear Poisson-type problems with F-Trefftz hybrid finite element model. *Eng Anal Bound Elem* 36: 39-46.
18. Wang H, Qin QH (2012) A new special element for stress concentration analysis of a plate with elliptical holes. *Acta Mechanica* 223: 1323-1340.
19. Wang H, Qin QH (2012) Numerical implementation of local effects due to two-dimensional discontinuous loads using special elements based on boundary integrals. *Eng Anal Bound Elem* 36: 1733-1745.