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Research Article

Dynamics Study of Compliant Mechanism with Damping

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ABSTRACT

Flexure mechanisms are used in many fields such as precision positioning, micro/nanoscale fabrication, force/torque sensors, optical fibers, bio-engineering, nano-imprint technology. Recently, there have been studies conducted about its mechanical characteristics so far and applications. However, most of these studies have just only focused on the static analysis or the inadequately dynamic analysis without considering the damping characteristics of flexure joints. This paper presents the fully dynamic analysis of a flexure mechanism with a combination of damping factors in particular. The mechanism provides small linear motion. The dynamic characteristics including the response of output link and the natural frequency of the flexure mechanism are determined based on a dynamic analysis on the pseudo-rigid- body diagram and based on the finite element model.

Keywords: Flexure; Pseudo-rigid-body diagram; Finite element analysis; Damping; dynamic

INTRODUCTION

If something bends to do what it is meant to do and also to accomplish something useful, then it is a compliant mechanism or flexure mechanism [1]. In recent decades, compliant mechanism has been a hot topic in precision engineering because it has many outstanding advantages that outweigh the rigid-body mechanisms for precision engineering applications including non-friction movements, no stick-slip behavior, no backlash, and smooth and highly repeatable motions, etc. As this sort of mechanism does not need to be lubricated periodically, it will be a very good choice for mechanical components operating in small and enclosed space or vacuum of conditions [2]. However, it still has several disadvantages due to its monolithic design such as limited displacement, so complex mechanical properties that need more studies to reduce the deviations between the analytical method and finite element method. Hence, there were many mathematical methods proposed to solve these "tough characteristics" of the compliant mechanism. When conducting any research related to a compliant mechanism, which is made up of flexure hinges, the first and most important thing to take into consideration is the relation between the stiffness and the displacement matrix of a flexure hinge. However, that connection almost depends on the type of material and geometry especially applied for a flexure. As a consequence, there were many mathematical models proposed to guide how to determine the deformation of compliant under external force and moment with reliable results, for instance: the model of Lobontiu et al. [3], for thin hinges. However, most recent studies have just only focused on investigating the state of deformation of compliant mechanism under static conditions whereas its applications may vary from static to dynamic conditions, therefore, this paper will illustrate the dynamic response of a compliant mechanism which is partially based on one of those stiffness matrices models.

There are two main categories of methods to approach a flexure system that are the distributed parameter and the finite element. For distributed parameter methods, it will be described by a system of partial differential equations [4]. Nevertheless, for complex systems, it is usually hard to find an equation describing the dynamic behavior of entire systems, so in these circumstances, approximate methods are further employed in order to simplify the problem. This article presents an approximate model called Pseudo-Rigid- Body (PRB Model) which was proposed in Dynamic Studies of Larry and Howell [5] to aid engineers in oscillation calculation. After the equivalent dynamic conversion, proceeding to build the second-order differential equation of motion for a compliant mechanism by using Lagrange's Equation (1) which was clearly and fully presented in Flexures of Stuart T. Smith [6].

$$\sum_{s=1}^{n}\left(\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right)-\frac{\partial T}{\partial q_s}+\frac{\partial R}{\partial \dot{q}_s}+\frac{\partial V}{\partial q_s}\right)=Q_s \qquad (1)$$

Where:

- T is the kinetic energy;
- R is the energy dissipated due to the damping characteristic;
- V is the potential energy;
- Q_s is the generalized force acting to produce a change in coordinate q_s.

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STATIC ANALYSIS OF FLEXURE MECHANISM

Design of flexure mechanism

As the research of Linß [4], there are approximate 12 types of different flexure. And its deformation may vary depending on the contour. However, this article only focuses on analyzing the performance of the circular hinges, one of the most popular type of flexure hinge. The static and dynamic response of a compliant mechanism which is made up of many flexure hinges will be analyzed and experimented to determine its response. The 3D and 2D design of the compliant mechanism will be presented in Figures 1 and 2.

Designing parameters will be chosen in the Table 1.

In which:

- $h=H-2R$ (mm)
- Applied material for entire mechanism: AISI 1045 Carbon Steel. Material properties:

– Young Modulus: E=205(GPa)

– Poisson's ratio: ν=0.29

- The width of mechanism: b=10 mm

There are many researches showing the way to determine the bending stiffness of the circular flexure hinges. One of those is the model of Lobontiu which will be applied to calculate the bending stiffness of the circular hinges in this paper. That model is expressed as followed function [3].

$$K_{\theta z}=\frac{Ebh^3(2R+h)(4H+h)^3}{24R}\left[h(4R+h)(6R^2+4Rh+h^2)+6R(2\mathrm{R}+\mathrm{h})^2\sqrt{h(4R+h)}\,arch\tan\sqrt{1+\frac{4R}{h}}\right]^{-1}$$

Based on the above function, I have the table of the bending stiffness value for each flexure as below in Table 2:

Using the Pseudo Rigid Body (PRB) theory of Larry L. Howell [5] to convert the compliant mechanism into a rigid body mechanism by replacing the flexure with a normal hinge and torsional spring, as shown in following Figure 3.

Under an arbitrary external force, the compliant mechanism will deform as Figure 4.

where:

- θ_3; θ_1: deflection angle.
- Y_3; Y_1: displacement in y direction.

Due to the minor deflection, the relation between Y and θ could be accepted approximately by the following equation: Y_3= $L\theta_3$; Y_1 = $L\theta_1$; θ_3 $3\theta_1$. So, the displacement will could be accepted approximately by the following equation: be fully determined if we can define the response of the deflection angle, θ_Z. However, the deflection angle depends on the external force (or moment) and the bending stiffness of the flexure hinge (Figures 3-6).

Determining the bending stiffness parameter by analytical method

Diagram of force and moment distribution and Geometric parameters described in Figures 7 and 8 below.

$$\sum M_A=-3aF_{21}+aF-M_1 \\ =-3aF_{21}+aF-K_1\theta_{Z1}=0 \quad (3)$$

$$F_{21}=\frac{F}{3}=-\frac{K_1\theta_1}{3a} \quad (4)$$

$$\sum M_D=M_3+M_{22}+M_{21}-aF_{12}=0 \\ K_3\theta_3+K_{22}\theta_{22}+K_{21}\theta_{21}=aF_{12} \quad (5)$$

Based on the Figure 8 to determine geometry parameters as below:

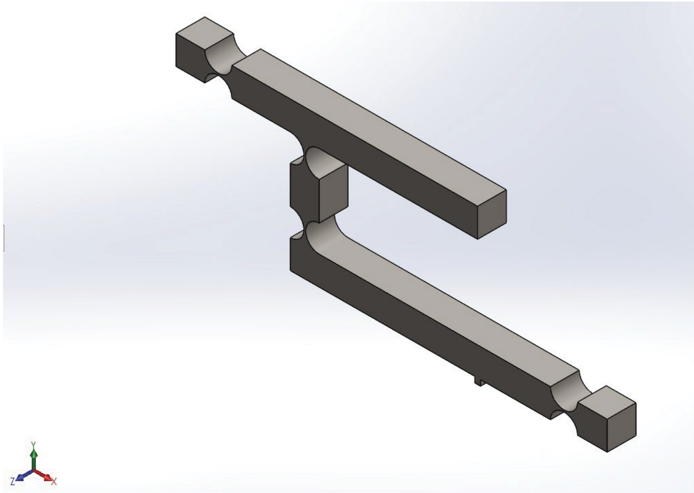

Figure 1: Design on solidworks.

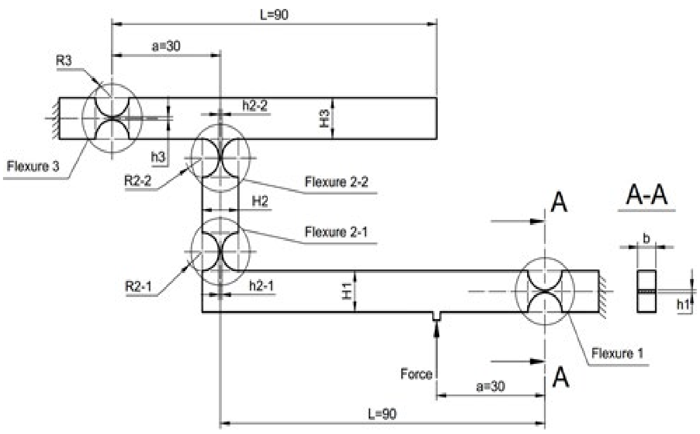

Figure 2: 2D design on Autocad.

Table 1: Designing parameters of the flexure mechanism.

No.	Parameters	Value	Unit	Note
1	H1	10	mm	Flexure 1
	h1	0.6		
	R1	4.7		
2	H2	10	mm	Flexure 2-1
	h2-1	0.6		
	R2-1	4.7		
	h2-2	0.6		Flexure 2-2
	R2-2	4.7		
3	H3	10	mm	Flexure 3
	h3	0.6		
	R3	4.7		

Table 2: The bending stiffness value for each flexure.

Description	Parameters	Value	Unit
Equivalent Momentum	Jeq=	$1,83.10^{-3}$	$kg.m^2$
	K_1	18961	
Bending stiffness	$K2-1=K2$	18961	$\frac{N\ m\ m}{rad}$
	$K2-2=K2$	18961	
	K_3	18961	
Equivalent bending stiffness	$K_{eq}=9K_3+12K_2+K_1$	18961	

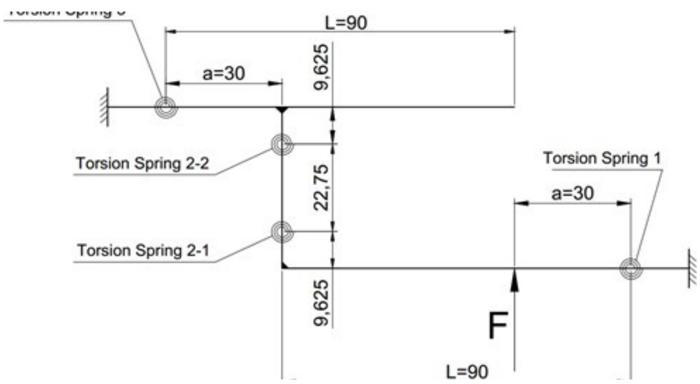

Figure 3: The compliant mechanism after converting by PRB model.

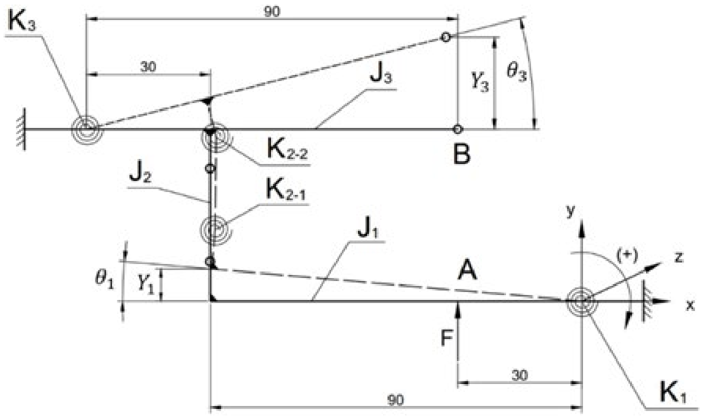

Figure 4: The PRB model deforms under an external force.

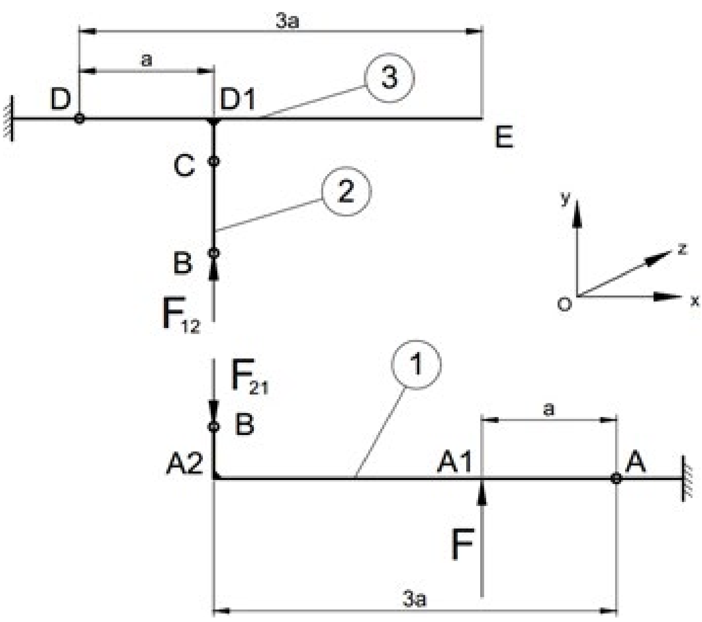

Figure 5: PRB model is divided into three parts.

$$\theta_{21} = \varphi_2 - \widehat{B} = \varphi_2 - BA_2A = \varphi_2 - \left(\frac{\pi}{2} - \theta_1\right) \quad (6)$$

$$\theta_{22} = \varphi_3 - \left(\frac{\pi}{2} - \alpha_3\right) - \varphi_2 \quad (7)$$

$$\varphi_3 = \pi - (\alpha_3 - \theta_3) \quad (8)$$

with: $Bx_{21}||A_2A$ and $x_{22'},x_{22}||Dx_3$ Substituting Eq. 5 with θ_{21}; θ_{22}; Ψ_3 from the Eq. 6; 7; 8 to get these equation:

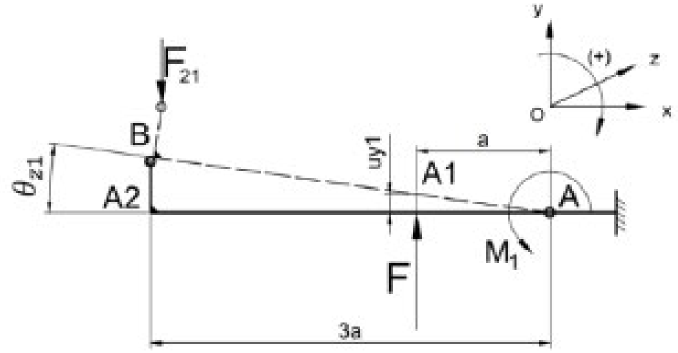

Figure 6: Diagram of force and moment distribution on part 1.

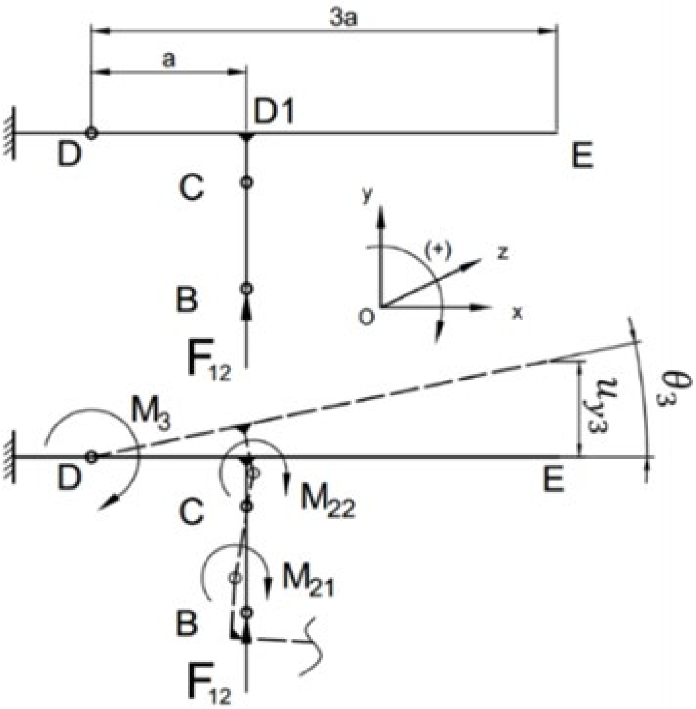

Figure 7: Diagram of force and moment distribution on part 2 and 3.

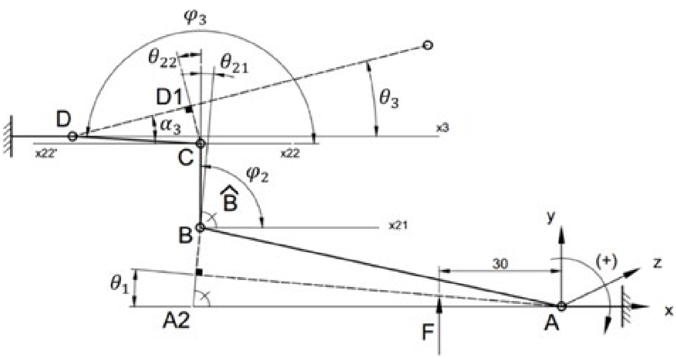

Figure 8: The geometry parameters when the mechanism deform.

$$3K_3\theta_1 + K_{22}\left[\varphi_3 - \left(\frac{\pi}{2} - \alpha_3\right) - \varphi_2\right] + K_{21}\left[\varphi_2\left(\frac{\pi}{2} - \theta\right)\right] = aF_{12} \quad (9)$$

$$\theta_1 = \frac{Fa}{9K_3 + 12K_2 + K_1} = \frac{\theta_3}{3} \quad (10)$$

We knew the link between the deflection angle and the torque is: $M=K\theta_Z$. Therefore, the equivalent bending stiffness of the entire mechanism is defined from Eq. 10:

$$K_{eq}=9K_3+12K_2+K_1 \quad (10)$$

DYNAMICS ANALYSIS OF FLEXURE

Mechanism

Setup of experiments: The experiment was set up; see in Figure 9, by utilizing a piezoelectric actuator, PAS015 of THORLABS, to excite the compliant mechanism to vibrate by applying a square pulse of voltage very quickly. It would lead to a result that the flexure mechanism will free vibrate. This paper mostly concentrates on evaluating the state of free vibration, without maintaining external force. Using the accelerometer and its display unit to measure the decrease of amplitude of the oscillation to determine the damping ratio in two different states, before and after the flexure hinges are covered with the damping material, which is represented in Figures 10 and 11.

Determining natural frequency: In order to obtain the natural frequency and natural modes of a system, un-damped free vibration equation is used because the damping has very little influence on the natural frequencies of a system [6,7]. Calculating the natural frequency then utilizing the module Modal in ANSYS software to verify the result.

Based on Figure 4, having the table of the parameters of flexure mechanism as below in Table 3.

The value of Momentum is given by Solidworks 2017. And the Natural frequency can be calculated as below:

$$f_{n-AM} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{J_{eq}}} \approx\sim 76Hz \quad (11)$$

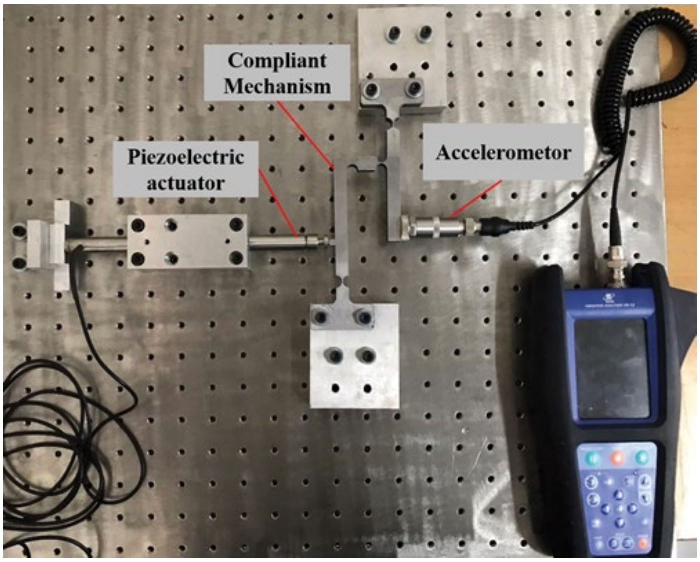

Figure 9: Setup for dynamic response measurement of damping ratio.


Figure 10: Flexure hinges before added with damping material.

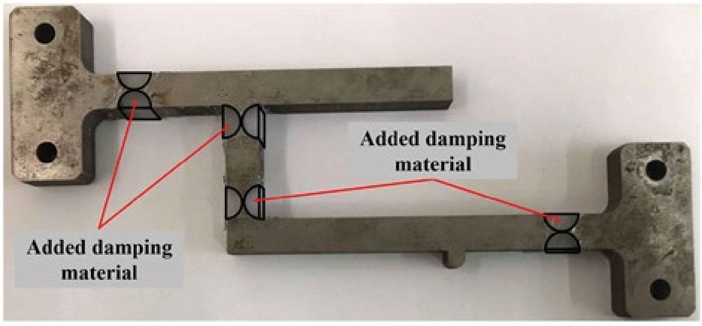

Figure 11: Flexure hinges after added with damping material.

Table 3: Dynamic parameters of the PRB model from Figure 4.

Description	Parameters	Value	Unit
Equivalent Momentum	J_{eq}=	$1, 83.10^{-3}$	$kg.m^2$
	K_1	18961	
Bending stiffness	$K2-1=K2$	18961	$\frac{Nmm}{rad}$
	$K2-2=K2$	18961	
	K_3	18961	
Equivalent bending stiffness	$K_{eq}=9K_3+12K_2+K_1$	18961	

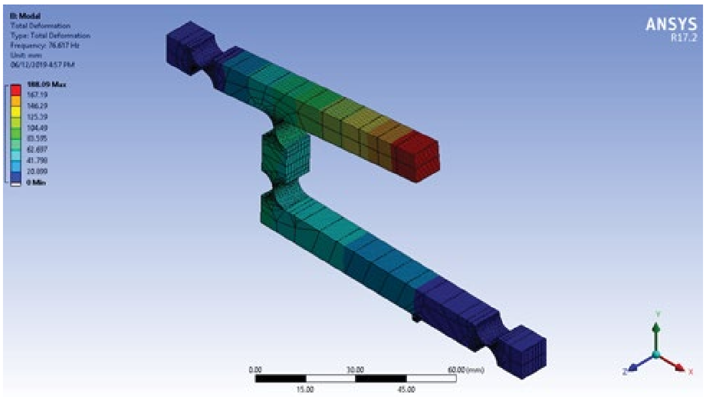

Figure 12: The natural frequency determined by ANSYS is fn-AS =76; 6Hz.

$$\omega_{n-AM} = 2\pi f_{n-AM} = 247, 17(rad/s) \quad (12)$$

And the result of ANSYS expressed in Figure 12 is approximately equal to the result calculating by Analytical Method (AM).

Dynamics of flexure with and without damping material

Damping measurement: There are various parameters to represent the damping characteristic such as specific damp- ing capacity, loss factor, Q-factor, and damping ratio, and models such as viscous, hysteretic, structural, and fluid. Before proceeding to measure damping in the compliant mechanism.

Researchers should take into consideration about what sort of models that could be applied properly to investigate the mechanical-energy dissipation properties in the compliant sys- tem. There are two common ways to measure the damping: time-response methods and frequency-response methods. It is basically different between these two methods that the first type uses a time-response record of the system to estimate damping while the second one uses a frequency-response record [8]. And, this paper will represent the model of viscous damping to estimate the damping ratio from the experiment. The Damping ratio is a parameter, usually denoted by ζ (zeta) provides a mathematical means of expressing the level of damping in a system relative to critical damping [8]. With the viscous damping model would lead to a concept which is logarithmic decrement, and denoted by δ, and ζ is a function of δ, which are given as [9]:

$$\varsigma = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{13}$$

$$\delta = \ln\frac{x(t)}{x(t+T)} \tag{14}$$

In this paper, the denotation: ζ_1 and ζ_2 are the Damping Ratio which indicate the state before and after supplementing the damping material (DM) respectively. Conducting experiments to determine Damping Ratio and the results are expressed in the Figures 13 and 14. Based on the data from experiments to determine the average value of Damping Ratio at two states.

$$\begin{cases} \overline{\varsigma_1} = \varsigma_1 = 0,0088 \\ \overline{\varsigma_2} = \varsigma_2 = 0,0142 \end{cases}$$

Dynamics equations: The Eq. 1 is the general dynamic equation of a motion system. Based on the Eq. 1 to derive the dynamic equation for the rotational coordinate system in this paper. And the displacement, velocity, and acceleration in direction Y will be given as: Displacement equation:

$$y(t) = Y_0 e^{-\zeta\omega nt}\cos(\omega_d t - \varphi_0) \tag{15}$$

Velocity equation:

$$v(t) = -Y_0\omega_n e^{-\varsigma\omega_n t}\left[\varsigma\cos(\omega_d t - \phi_0) + 2\varsigma\sqrt{1-\varsigma^2}\sin(\omega_d t - \phi_0)\right] \tag{16}$$

Acceleration equation:

$$a(t) = Y_0\omega^2{}_n e^{-\varsigma\omega_n t}\left[(2\varsigma^2 - 1)\cos(\omega_d t - \phi_0) + 2\varsigma\sqrt{1-\varsigma^2}\sin(\omega_d t - \phi_0)\right] \tag{17}$$

in which:

$$\omega_n = \sqrt{\frac{K_{eq}}{J_{eq}}} \text{ :natural angular frequency.}$$

$$\omega_d = \omega_n\sqrt{1-\varsigma_2} \text{ :damped angular frequency.}$$

Evaluating the dynamics equations results by ANSYS: Converting the physical model of the flexure mechanism into the rigid body model in ANSYS to analyze the dynamics characteristic of the flexure system (Figure 15).

With this module, it is required to provide the data input including the torsional stiffness and the damping coefficient, and it is important to ensure the equivalent momentum of the entire mechanism. With the damping coefficient, it could be derived from the damping ratio and the critical damping coefficient by the formula as below:

$$\varsigma = \frac{c}{c_c}$$

In which $c_c = 2\sqrt{K_{eq}J_{eq}} = 2K_{eq}\omega_n$ (18)

RESULTS AND DISCUSSION

In this paper, I will only plot the velocity graph because the displacement and acceleration equation can be preceded in a similar way. These figures below show the results of the dynamics equation in terms of velocity in y direction. Also, the results comparison between the analytical and ANSYS analysis method. In Figure 15, it is easy to observe that there is a minor error between two

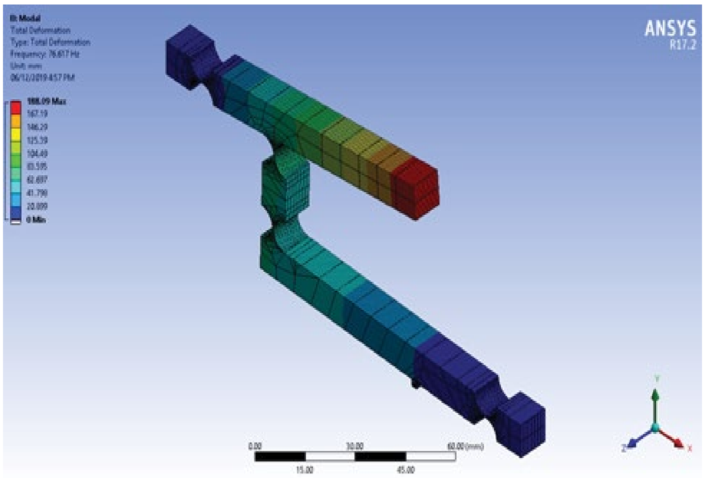

Figure 12: The natural frequency determined by ANSYS is fn-AS =76; 6Hz.

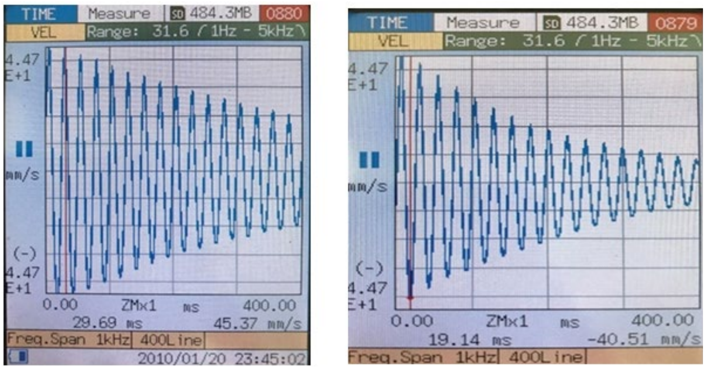

Figure 13: Obtained vibration graph from the accelerometer.

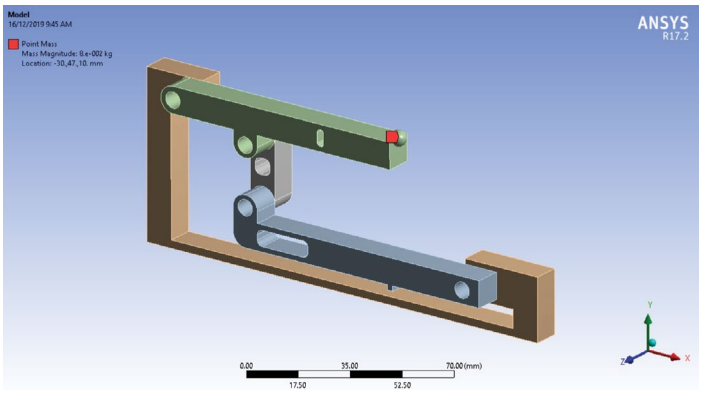

Figure 14: Equivalent convert to module rigid dynamics in ANSYS.

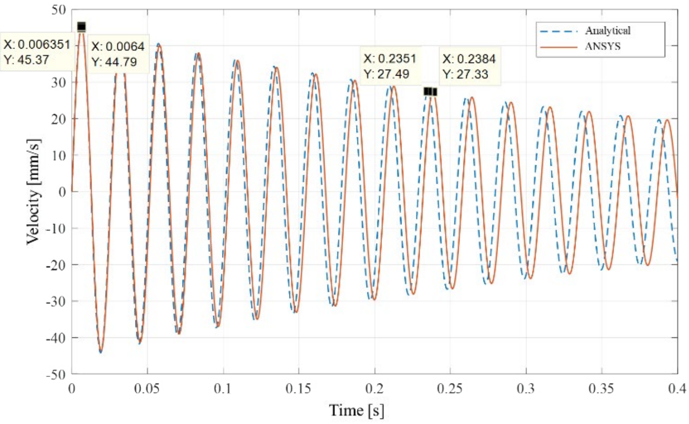

Figure 15: % error between analytical and ANSYS method at ς_1 case.

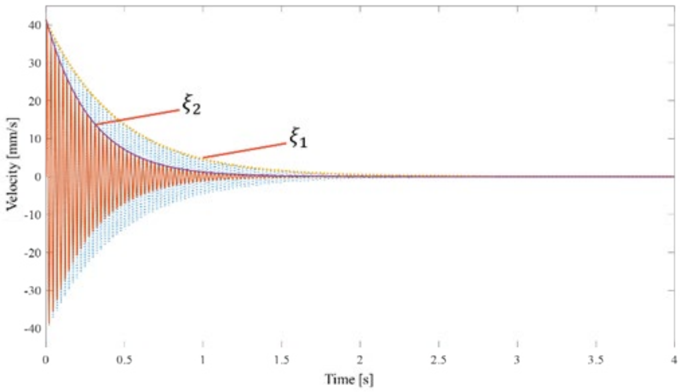

Figure 16: Free vibration response of entire mechanism at two state, ς_1 and ς_2.

methods in the time domain of 0.4 second. However, it exists an accumulative time error, and author proceeded to expand the time domain to more than 1 second to estimate the accumulative time error, and it will be reconsidered at 2 condition, ζ_1 and ζ_2, in the Tables 4 and 5 this error would be an acceptable result. In Figure 16, it is very optimistic to observe the result that the supplementary material reduced the time of free vibration and the amplitude of vibration at the same time comparing to the condition of no damping material. From the collected data in the Tables 4 and 5, it is observed that %velocity error between analytical and ANSYS is minor. And, the accumulative timer error is a big gap in the 32^{nd} period of vibration, that error has a tendency to keep increasing if damped frequency of this mechanism is f_d=39Hz (this the time domain is expanded. However, considering that the frequency was recalculated due the effect of accelerometer mass), so if collecting the data in 2 seconds, it means that the total accumulative time error may be over 100% - over a period of vibration. And in 2 seconds the mechanism vibrated nearly 78 periods, therefore, the actual error will be 1/78=1,28%, this error would be an acceptable result.

Table 5: Received data at ζ_2, with damping material.

Period of Vibration		1	10	32
Time (s)	ANSYS	0, 00635	0, 2351	0, 9724
	AM	0, 0064	0, 2384	0, 9862
Velocity (mm/s)	ANSYS	45, 37	27, 49	5, 474
	AM	44, 79	27, 33	5, 567
% Time error vs. period of vibration, T_d = 0, 0254 (s)		0, 34%	12, 99%	53, 54%
% Velocity error		1, 29%	0, 59%	1, 67%

CONCLUSION

This paper established a method to reduce the time of free vibration by adding a damping material. It showed a positive tendency to enhance a higher accuracy in dynamic control for other compliant mechanism systems. Establishing a method to determine the dynamic equation by applying the damping ratio in the equation to predict its transformation under a certain condition. The data analysis between analytical and ANSYS method bring a positive signal about the reliability of the measurement and theory. Module Rigid Dynamics demonstrates its advantages in solving the dynamic problems of flexure mechanism which are usually required a huge resource from the computer.

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