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Dynamic Systems for Signaling Parasite-Host from Differential Equations

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Abstract

Based on the stability theory of fractional order differential equations, the adaptive anti-synchronization of some fractional-order differential equations for modeling parasite-host are studied and their approximate solutions are presented. Combining the adaptive control method, the adaptive anti-synchronization, Lyapunov equations and parameter identification of some fractional-order differential equations are realized by designing suitable controllers and parameter adaptive laws. Finally, numerical simulations further demonstrate the feasibility and validity of this method.

Keywords: Developmental Biology; Fractional-order; Self-adaption; Robust anti-synchronization; Lyapunov equations; Numerical simulations

Introduction

Mathematical models, using ordinary differential equations with integer order, have been proven valuable in understanding the dynamics of biological systems. However, the behavior of most biological systems has memory or aftereffects. The modelling of these systems by fractional-order differential equations has more advantages than classical integer-order mathematical modeling, in which such effects are neglected. The topic of fractional calculus (theory of integration and differentiation of an arbitrary order) was started over 300 years ago. Recently, fractional differential equations have attracted many scientists and researchers due to the tremendous use in Mathematical Biology. The reason of using fractional-order differential equations (FOD) is that FOD are naturally related to systems with memory which exists in most biological systems. Also they are closely related to fractals which are abundant in biological systems. The results derived of the fractional system are of a more general nature. Respectively, solutions of FOD spread at a faster rate than the classical differential equations, and may exhibit asymmetry. Theory of differential equations in the formation process, the solution of differential equations there have been many methods, such as separation of variables, variable substitution method, constant variation, and integral factor method. Especially the integral factor method is the latest and the biggest role which is the essence of differential equation into appropriate solution which is easy to draw, so we can easily obtain the solution of differential equations. Therefore, the integral factor method is the key to solution of different equations.

Fractional calculus is the extension of the standard calculus with integer order, which researches on the theory and application of the differential and integral nonstandard operators of arbitrary order. It is an important branch of mathematical analysis. With the emergence of many fractal dimension facts in the nature and science, fractional calculus theory and fractional differential equations have got more confirmations of mathematicians and attracted more attention in Mathematical Biology [1-5].

Model Formulation

Self-adaptive robust anti-synchronization of some fractional-order differential equations

First, we consider the following fractional-order differential equations as one system:

$$\begin{cases} \dfrac{d^{\alpha_1} x_1}{dt^{\alpha_1}} = \alpha(x_2 - x_1) + x_4 \\ \dfrac{d^{\alpha_2} x_2}{dt^{\alpha_2}} = hx_1 - x_1x_3 + cx_2 \\ \dfrac{d^{\alpha_3} x_3}{dt^{\alpha_3}} = x_1x_2 - bx_3 \\ \dfrac{d^{\alpha_4} x_4}{dt^{\alpha_4}} = x_2x_3 + rx_4 \end{cases} \tag{1}$$

where $0 < \alpha_i < 1, (i = 1,2,3,4)$ is a parameter describing the order of the system, x_i $(i = 1,2,3,4)$ is the anti-synchronization function of the time t. If a=35, b=3, c=12, h=7, $0.085 \le r \le 0.798$, then the system is in a chaotic state.

Next, suppose that some ractional-order differential equations are response systems:

$$\begin{cases} \dfrac{d^{\beta_1} y_1}{dt^{\beta_1}} = \hat{\alpha}(y_2 - y_1) + y_4 + u_1(t) \\ \dfrac{d^{\beta_2} y_2}{dt^{\beta_2}} = \hat{h}y_1 - y_1y_3 + \hat{c}y_2 + u_2(t) \\ \dfrac{d^{\beta_3} y_3}{dt^{\beta_3}} = y_1y_2 - \hat{b}y_3 + u_3(t) \\ \dfrac{d^{\beta_4} y_4}{dt^{\beta_4}} = y_2y_3 + \hat{r}y_4 + u_4(t) \end{cases} \tag{2}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{h}, \hat{r}$ are the estimated values of the parameters a, b, c, h, r on the system (1) respectively; and $0 < \beta_i < 1, (i = 1,2,3,4)$

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is a parameter describing the order of the system (2); $d^q/dt^q = D_t^q, q = \alpha_i, \beta_i, (i = 1,2,3,4)$ are in the Caputo sense, and $x = (x_1(t), x_2(t), x_3(t), x_4(t))^T$, $y = (y_1(t), y_2(t), y_3(t), y_4(t))^T$ are the status vectors of system (1) and system (2) respectively. $u(t,x,y) = (u_1(t), u_2(t), u_3(t), u_4(t))$ is the controller.

If $\alpha_i = 0.98, (i = 1,2,3,4), a = 35, b = 3, c = 12, h = 7, r = 0.5$, then the diagram of the attractors of system (1) can be seen in Figure 1.

Controller design

According to the definition of robust anti-synchronization error, suppose that the robust anti-synchronization error is $e = x + y$. If for any $x(0), y(0)$ satisfy the condition $\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|x(t)+y(t)\| = 0$, then we say that system (1) and system (2) achieve robust anti-synchronization.

On the basis of adaptive control methods, we can give the design of the controller:

$$\begin{cases} u_1(t) = -D_t^{\beta_1}x_1 - D_t^{\alpha_1}x_1 - (\hat{a}(y_2 - y_1) + y_4) + \hat{a}(x_2 - x_1) + x_4 - k_1e_1 \\ u_2(t) = -D_t^{\beta_2}x_2 - D_t^{\alpha_2}x_2 - (\hat{h}(y_1 - y_1y_3 + \hat{c}y_2) + \hat{h}x_1 - x_1x_3 + \hat{c}x_2 - k_2e_2 \\ u_3(t) = -D_t^{\beta_3}x_2 - D_t^{\alpha_3}x_2 - (y_1y_2 - \hat{b}y_3) + x_1x_2 - \hat{b}x_3 - k_3e_3 \\ u_4(t) = -D_t^{\beta_4}x_2 - D_t^{\alpha_4}x_2 - (y_2y_3 + \hat{r}y_4) + x_2x_3 + \hat{r}x_4 - k_4e_4 \end{cases} \quad (3)$$

where $e_1 = x_1 + y_1, e_2 = x_2 + y_2, e_3 = x_3 + y_3, e_4 = x_4 + y_4$, $k_i > 0, (i = 1,2,3,4)$. If $t \to \infty$, then $\|e\| \to 0$,and system (1) and system (2) achieve robust anti-synchronization.

If we put (3) and system (1) to system (2) , then the following error equations can be obtained between the groups for some fractional differential equations:

$$\begin{cases} D_t^{\beta_1}e_1 = -e_a(x_2 - x_1) - k_1e_1 \\ D_t^{\beta_2}e_2 = -e_dx_1 - e_cx_2 - k_2e_2 \\ D_t^{\beta_3}e_3 = e_bx_3 - k_3e_3 \\ D_t^{\beta_4}e_4 = -e_rx_4 - k_4e_4 \end{cases} \quad (4)$$

where $e_a = a - \hat{a}, e_b = b - \hat{b}, e_c = c - \hat{c}, e_h = h - \hat{h}, e_r = r - \hat{r}$ are the parameter estimation errors.

Next, according to (4), we design the adaptive update law for each parameter estimation error:

$$\begin{cases} D_t^{\beta_5}e_a = (x_2 - x_1)e_1 \\ D_t^{\beta_6}e_b = -x_3e_3 \\ D_t^{\beta_7}e_c = x_2e_2 \\ D_t^{\beta_8}e_h = x_1e_2 \\ D_t^{\beta_9}e_r = x_4e_4 \end{cases} \quad (5)$$

where $0 < \beta_i < 1, \; (i = 5,6,7,8,9)$.

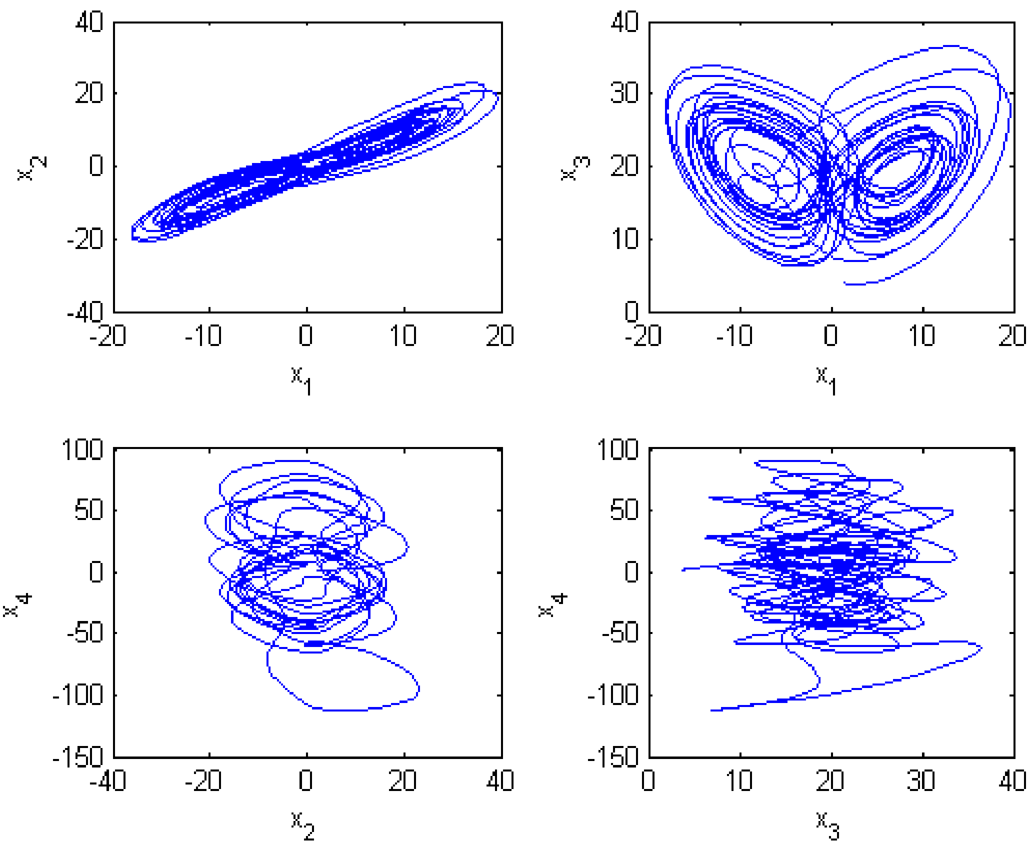

Figure 1: Diagram of the attractors.

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According to $e_a = a - \hat{a}, e_b = b - \hat{b}, e_c = c - \hat{c}, e_h = h - \hat{h}, e_r = r - \hat{r}$ and (5), we can get the parameters of the adaptive control law:

$$\begin{cases} D_t^{\beta_5}\hat{a} = (x_1 - x_2)e_1 \\ D_t^{\beta_6}\hat{b} = x_3 e_3 \\ D_t^{\beta_7}\hat{c} = -x_2 e_2 \\ D_t^{\beta_8}\hat{h} = -x_1 e_2 \\ D_t^{\beta_9}\hat{r} = -x_4 e_4 \end{cases} \quad (6)$$

According to (4) and (5), we get the total error of the system:

$$D_t^{\beta} E = AE \quad (7)$$

where

$D_t^{\beta} E = (D_t^{\beta_1} e_1, D_t^{\beta_2} e_2, D_t^{\beta_3} e_3, D_t^{\beta_4} e_4, D_t^{\beta_5} e_a, D_t^{\beta_6} e_b, D_t^{\beta_7} e_c, D_t^{\beta_8} e_h, D_t^{\beta_9} e_r)^{\mathrm{T}}$,

$E = (e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_h, e_r)^{\mathrm{T}}, 0 < \beta_i < 1, (i = 1, \cdots, 9)$.

Then we consider Eq. (7), and expand the formula, we obtain:

$$D_t^{\beta}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_a \\ e_b \\ e_c \\ e_d \\ e_r \end{pmatrix} = AE = \begin{pmatrix} -k_1 & 0 & 0 & 0 & -(x_2 - x_1) & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 & 0 & -x_2 & -x_1 & 0 \\ 0 & 0 & -k_3 & 0 & 0 & x_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 & 0 & 0 & 0 & -x_4 \\ x_2 - x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -x_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_a \\ e_b \\ e_c \\ e_d \\ e_r \end{pmatrix}$$

Setting $P = E_9$. Then we obtain the following result:

$$\begin{aligned} & AP + PA^T \\ & = A + A^T = -Q \\ & = \begin{pmatrix} -2k_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2k_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (8)$$

where $k_i > 0, (i = 1, 2, 3, 4), Q = diag(2k_1, 2k_2, 2k_3, 2k_4, 0, 0, 0, 0, 0)$.

It is easy to see that $Q = diag(2k_1, 2k_2, 2k_3, 2k_4, 0, 0, 0, 0, 0)$ is a semi-positive definite matrix. Then, the state variable of (7) $E = (e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_h, e_r)^{\mathrm{T}}$ is asymptotically stable, that is, $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_h, e_r$ approach zero asymptotically with time. Therefore, we achieve a number of adaptive robust set of fractional differential equations anti-synchronization.

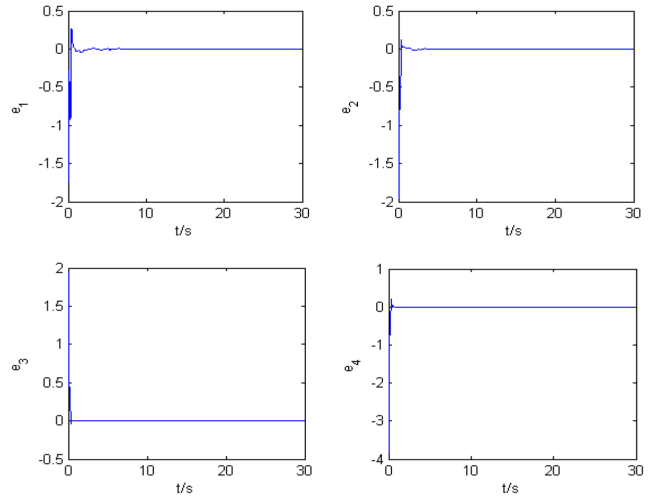

Figure 2: Robust anti-synchronization error e_1, e_2, e_3, e_4 curve between system (1) and system (2).

Numerical simulation

In order to verify the effectiveness of the methods shown above, the time step is taken as $h = 0.0025s$, and $T = 30s$, $\alpha_i = 0.98, (i = 1, 2, 3, 4)$, $\beta_i = 0.96, (i = 5, \cdots, 9)$, a=35, b=3, c=12, h=7, r=0.5, the time taken for the simulation, the order taking system, to select the system parameters, the initial state value of system (1) is $x(0) = (2, 0, 1, 1)$, the initial state value of system (2) is $y(0) = (-4, -2, 1, -5)$. Therefore, the robust anti-synchronization error curve between system (1) and system (2) is shown in the following Figure 2.

Conclusion

Our theoretical results have been validated with corresponding numerical simulations and can be used as a good model for signaling parasite-host. The numerical simulations also confirm the advantages of the mathematical tools using fractional-order differential models in biological systems.

Acknowledgements

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