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Dynamic Analysis of Cam Manufacturing

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Abstract

In cam milling process, cutting force is a variant factor during every time period and cam has a quite complex profile that leads to alternate force direction. These consequently, create machine vibration. The dynamic behaviour of machine can be predicted approximately if it is represented by a mathematical model. This paper shows result of cam cutting machine's dynamic, which used Lagrange's equation to solve. In this case, the machine vibration is surveyed only dimensions such as X and Y through using cutting condition with alloy cutting tool to mill a 10 mm thickness steel cam. The machine is modelled into the two degree of freedom vibrating system follow X and Y direction. Each of X and Y table equal to the compound: stiffness, damper and mass, which applied as constant coefficients in Lagrange's equation. On the other hand, analysing cam characteristic and milling process in detail provides the resultant cutting follow X and Y in order to become external force of previous equation. After giving data in sufficient that necessary for problem, Matlab Simulink displays the vibration of X, Y for two states tangent force factor $K_t=299.3$ and $K_t=598.6$. At the end, it gives a comparison between these states.

Keywords: Cam mechanism; Milling; Dynamics

Introduction

A cam - a part of the cam - follower mechanical system, which complex peripheral profile depends on follower movement rule is an important mechanism and used popular in automatic mechanical. Designers always expect the accuracy position of follower motion during all operating periods. It means that cam machining has to achieve a tolerance deviation. Vibration of cutting machine has a significant influence on affecting the success of machining. As an illustration, a high value cutting force ease to create a large amplitude of fluctuation that of course decrease the surface quality and dimensional tolerance [1-8]. Moreover, vibration causes the claim and machine duration to be harmful. As a result, the machine dynamic should be considered studying like a main manufacturing factor to handle issues above [9-17]. There are several documents refer to cam machining. For instance, Rothbart [6] illustrated cam manufacturing method, tolerance and errors. In addition, Altintas [17] represent the mechanics of metal cutting. Stephen and Radze [14] mentioned kinematic geometry of surface machining in 2014. They play a role just like the basic theory for this search. The paper also supplies some new acknowledge such as improving the cutting force formulations and analysing the cam machine dynamic, which others do not yet. Furthermore, it provides the essential data to evaluate and choose suitable cutting condition in order to increase the accuracy of cam.

Mathematical Modelling

Physical model the dynamic of cam machining

A cam machine model is explored, include table X carries milling cutting tool and table Y bears table X, both of them can only move one direction which same their name (Figure 1). Tool has clockwise spins around its axis and cam peripheral in the chosen cutting condition. In contrast, cam is fixed stationary. In mathematical model, tables X–Y become mass (m₁, m₂), damping (c₁, c₂) spring (k₁, k₂) and displacements (x, y). Likewise cutter gives the information of cutter's diameter (d) feed rate (f) (or cutting centre velocity V_{B1}), angular velocity ω_c the resultant cutting force F_c . Last of all, follower-cam displays S: follower displacement (S= (ϕ)) follower' angle rotation $\phi=\phi$ (t), follower offset e, roller's diameter (D), cutting icentre velocity, tangent velocity and transitive velocity of follower (v_c, v_c) (Figure 2).



As mention above, machine's kinematic depends on cam profile, so firstly it needs to determine some factors relative (Figure 2). Cam profile coordinates B at cam angle ϕ :

Ix coordinate of cam surface profile: .x_B=x=ecosφ-Ssinφ

y coordinate of cam surface profile: $y_B = y = esin\phi + Scos\phi$

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Figure 2: Analyze the kinetic of cam machining.



Figure 3: Cutting dynamic model.



Radius of the roller center's curvature s at B:

 $\rho = [e^2 + s^2 + S^2 + 2eS]^{3/2} / [e^2 + s^2 + 2S^2 + 3eS' + SS'']$ (1)

.Cam profile radius at B:

$$\rho_0 = \rho - \frac{D}{2} = \left[e^2 + S^2 + S^2 + 2eS \right]^{3/2} / |e^2 + S^2 + 2S^2 + 3eS + SS|$$
(2)

Profile radius of milling cutting centre B₁:

$$\rho_{1} = \rho - \frac{D - d}{2} = \left[e^{2} + S^{2} + S^{2} + 2eS \right]^{3/2} / |e^{2} + S^{2} + 2S^{2} + 3eS + SS| - \frac{D - d}{2}$$
(3)

The pressure angle will be: $\alpha = \tan^{-1}[(S'-e)]/S$ (4)

The velocity of milling cutting centre: $\vec{v}_{B1} = \vec{v}_c + \vec{v}_s$ (5)

Therefore, follower's angular velocity at t is:

$$\omega = \mathbf{v}_{\rm B1} \cdot \cos \alpha / \left(\mathbf{S} - \frac{D+d}{2} \cos \alpha \right) \tag{6}$$

Denominate $\beta = (AB_1, OX)$ determinated by: $\beta = 2\pi - \pi/2 - \alpha - (\pi - \phi) = \pi/2 - \alpha + \beta$

Cam centre coordinate at A:

x coordinate of cam surface profile: $x_{A} = e\cos\phi - S\sin\phi - \rho\sin(\alpha - \phi)$

y coordinate of cam surface profile: $y_A = esin\phi + Scos\phi - \rho cos(\alpha - \phi)$

Dynamic of milling processing

Milling cutters can be considered to have two orthogonal degrees of freedom as shown in Figure 3. The cutter is assumed to have number of teeth with a zero helix angle. The cutting forces excite the structure in the radial force F_r and tangential force F_t , causing displacement X and Y. The dynamic displacements are carried to rotating tooth in the radial or chip thickness direction with the coordinate transformation of fdx.sin $(\phi_1+\tau)+dy.cos(\phi_1+\tau)$ where , is

the instantaneous angular immersion of tooth measured anticlockwise from the cutting edge starts to contact work piece and τ =sin⁻¹ (0.5s/r). The resulting chip thickness consists of a static part h, attributed to rigid body motion of the cutter, and a dynamic component caused by the vibrations of the tool at the present and previous tooth periods. Because the chip thickness is measured in the radial direction, the total chip load [17] can be expressed by

$$h(t) = \{h + dxsin(-\tau) + dycos(\phi_i - \tau)\}g(i)$$
(7)

The function $g(\phi_j)$ is a unit step function that determines whether the tooth is in or out of cut, that is

$$\begin{cases} g(\phi_j) = 0 \text{ if } \phi_j < \phi_{st} \text{ } or \phi_{ex} < \phi_j \\ g(\phi_j) = 0 \text{ if } \phi_{st} \le \phi_j \le \phi_{ex} \end{cases}$$

The static chip load h (7) divided into 2 period:

If
$$\phi_{st} \le \phi_c \le \phi_{st} + \tau + \eta$$
, then

h=s{cos(τ - ϕ_1)tan[0.5sin⁻¹ $\langle (\tau-\phi_1)/r \rangle \rangle$]-sin(τ - ϕ_1)} (8)

If, $\varphi_{st} + \tau + \eta < \varphi_c \le \varphi_{st} + \tau + \eta + \upsilon$ then

h=rsin
$$(\upsilon + \tau - \varphi_1)$$
 [tan $\langle (\upsilon + \tau - \varphi_1)/2 + tan (\varphi_1 - \tau)$]. (9)

Where $\eta = \tan^{-1}[\sqrt{2rb-b^2-s}]/(r-b)v = \cos^{-1}[(r-b)/r]$ is the angle between _perpendicular direction work plane with h_{max} and out of cutting area. In addition, start angle:

 φ_{st} =- $\pi/2+\tau$, cutting angle: φ_{st} =- $\pi/2+\tau-\varphi_1$, φ_{ex} =- $\pi/2-\eta$, exit angle: and φ_p =2 π/Z , .cutting pitch angle:

When mill on plane, the cutting force includes tangent and radial cutting force F_{t} , F_{r}

$$F_{t}=K_{t}th=K_{t}t \left\{h+dxsin\left(\varphi_{1}-\tau\right)+dycos(\varphi_{1}-\tau)\right\}g(1)$$
(10)

$$F_{r}=K_{r}th=K_{r}t \{h+dxsin(\phi_{1}-\tau)+dycos(\phi_{1}-\tau)\}g(\phi_{1})$$
(11)

Project F_{t} (10), F (11) into X and Y direction get F_{x} , F_{y}

$$F_{x} = F_{t} \cos \left[\tau - \varphi_{j}\right] - F_{r} \sin \left[\tau - \varphi_{j}\right]$$
(12)

$$F_{v} = F_{t} \cos[\tau - j] - F_{r} \sin[\tau - \varphi_{i}]$$
(13)

Apply (12), (13) into cam machining in Figure 4, X and Y direction force become

$$F_{x} = F_{t} \cos \left[\phi - \alpha + \tau - \phi_{j}\right] - F_{r} \sin \left[\phi - \alpha + \tau - \phi_{j}\right]$$
(14)

$$F_{y} = F_{t} \cos \left[\phi - \alpha + \tau - \phi_{j}\right] - F_{r} \sin \left[\phi - \alpha + \tau - \phi_{j}\right]$$
(15)

Name	Value	Unit
Cam characteristic		
Follower offset e	40	mm
Roller diameter D	40	mm
Cam thickness (steel) t	10	mm
Machine parameter		
Spring k ₁ , k ₂	537728, 296881	N/mm²)
Damping c ₁ , c ₂	5	Ns/mm
Mass m ₁ , m ₂	70, 100	kg
Cutting condition (up-milling)		
Cutter diameter (alloy) d	20	mm
Main spindle speed n	660	rev/m
Feed rate s	0.18	
Feed rate (F= nSZ)	3.96	mm/m
Thickness	10	mm
Number of teeth Z	2	tooth
Resultant cutting force F _{cmax}	383.1	N

Table 1: The value input.





Dynamics of cam cutting

To survey the oscillation of this machine, Lagrange's equation [2] is compatible to use

$$\frac{\mathrm{d}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}} \frac{\partial V}{\partial q} - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = F$$
(16)

Page 3 of 5

F is the total external force, T [2] and V [2] are the system kinetic and potential energy, respectively. They are defined in (17) and (18)

$$T = \frac{1}{2} \sum_{l=3}^{0} [m l(x_{l}^{\prime 2} + y_{l}^{\prime 2}) + J_{l} \phi_{l}^{2}]$$
(17)

$$V = \sum [m_i g y_i + 0.5 (\Delta l)^2]$$
⁽¹⁸⁾

Combine equations (10) and (11) with equations (16-18) to derive the dynamic equation of cam machining (19) and (20):

$$m_{1}x''+c_{1}x'+k_{1}x=F_{t}\cos\left[\phi-\alpha+\tau-\phi_{j}\right]-F_{r}\sin\left[\phi-\alpha+\tau-\phi_{j}\tau-\phi_{j}\right]$$
(19)

$$m_2 y'' + c_2 y' + k_2 y = F_t \sin \left[\phi - \alpha + \tau - \varphi_i\right] - F_r \cos \left[\phi - \alpha + \tau - \varphi_i \tau - \varphi_i\right]$$
(20)

Example Model

The input value includes machine parameters in Table 1 and follower displacement rule [4] in Figures 5 and 6.

Use the data in Table 1, the follower angle rotation to (21), (22), thus force factor K_{i} , K_{r} and follower angular rotation are:

$$\omega = F\cos\alpha / (S - \frac{D+d}{2}\cos\alpha) = 3.96\cos\alpha / (S - 30\cos\alpha)$$
(21)

$$K_t = F_c / th(t)_{max} = 299, K_r = 0.3K_t = 89.3$$
 (22)

Derive milling tool angle velocity $\omega_c = 22\pi$ (rad/s) $0 \le \varphi_1 \le \cos^{-1} 0.7$ so

the so cutting time $t_c = \frac{\Phi_{jmax}}{\omega_c} = 0.01115(s)$ cutting time:

$$t_{p}=2\pi/(Zw_{c})=1/22$$
 (s)

Finally, model dynamic equation will become (23) and (24), then they are solved by Matlab Simulink similarly.

 $\begin{array}{ll} 200x''+0,002x'+1000000x=10\{h+dxsin \ (\phi_j-\tau)+dycos \ (\phi_j-\tau)\}g \ (\phi_j) \\ \{299.3cos \ (\varphi-\alpha+\tau-\phi_i)-89.8sin \ (\varphi+\tau-\phi_i)\} \ (23) \end{array}$

 $\begin{array}{ll} 100y' + 0,002 + 350000y'' = 10\{h + dxsin \quad (\phi_j - \tau) + dycos \quad (\phi_j - \tau)\}g & (\phi_j) \\ \{299.3cos \ (\varphi - \alpha + \tau - \phi_i \) - 89.8sin \ (\varphi - + \tau - \phi_i)\} \end{array}$

Result and Discussion

Figures 7-11 show the table X and Y oscillation in two different stations: k_t =299.3 and k_t =598.6 during one cutting period (173.85s).

Natural circular frequency [1,2]:

$$\omega_{n1} = \sqrt{k_1/m_1} = 1624.5 (rad/s)$$
(25)

$$\omega_{n2} = \sqrt{k_2/m_2} = 2059.4 (rad/s)$$
(26)

Damping ratio,

$$\zeta_1 = 0.5c_1 / \sqrt{k_1 m_1} = 0.00034 < 1; \\ \zeta_1 = 0.5c_2 / \sqrt{k_2 m_2} = 0.00055 < 1$$
(27)

. Therefore, with the aid of Euler's formula, the general solution x(t) and y(t) can be written in the form underdamping free vibration [1], derives the freedom vibrating equation of machine.

 $X=0.0023e^{-0.55t}sin [1624.5t] (mm)$ (28)

$$Y = 0.0027 e^{-1.13t} \sin \left[2059.4t \right] (mm)$$
(29)



Figure 8: X table oscillate in the first second and one period (173.85 s) by $K_{t\pm}598.6.$



Figure 9: Cutting force diagram by K_{t=}598.6.



Figure 10: Y table oscillate in the first second and one period (173.85 s) by $K_{\rm t}{=}299.3.$



Force F_x , F_y in Figures 6 and 8 is the result of the discontinuous cutting of milling process. They also have the variant value during each tool's revolution.

The results show the relationship between cutting force and oscillation. Assume that cutting force increase 150%, yet other conditions still unchanged. As anticipated, this boost as two times as much as the amplitude of fluctuation and displacement of X, Y. In other words, the amplitude of X change from 1.5×10^{-4} (mm) to 3×10^{-4} (mm) and other is 3.5×10^{-5} (mm) to 0.7×10^{-5} (mm). The maximum

X and Y displacements also have the same trend, those rise from 3.1 \times 10⁻⁴, 3 \times 10⁻⁴(mm) to 6.2 \times 10⁻⁴ (mm), 6 \times 10⁻⁴ (mm) sequentially. Their shapes alternate between F_x and F_y In contrast, the frequency responses are invariable, that means the input load and the table's physical characteristic almost affect to the table's movement. Therefore, the vibration is reduced quickly by increasing the stiffness of cutting system: tool, clamp, machine, machine's operating tightness. The rising mass and damper of tables benefit for machining in the same way.

In this situation, cam cutting machine's factors and the hardness of cam work piece are considered being unchanged. To keep machining process smoothly, it needs to decrease cutting force. Feed rate, cutting speed, thickness and width have the influence on cutting force. Tool material and abrasive cutter edge are in the same way. Those have to be controlled in strictness. The results are nearly similar as real milling models such as shoulder milling. It is useful for estimating the machining deviation.

Conclusion

The cam machining is modelled on elements: mass, springs, dampers of Lagrange's equation. Also, milling force, external force of previous equation, is also analysed thoroughly. Then the machine's vibration is completely achieved by using Matlab Simulink to solve Lagrange's equation. The results illustrate explicitly the displacements and frequencies of machine' table, those are correspondent with a rigid cutting system. Finally, this paper is a beneficial result to study the decline of cam machining deviation. In future, this model should be developed in real testing model to have more completely the evaluation of the dynamic of cam cutting machine.

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Page 5 of 5

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