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# Determination of Temperature Distribution for Porous Fin with Temperature-Dependent Heat Generation by Homotopy Analysis Method

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## Abstract

In this study, highly accurate analytical methods, Homotopy Analysis Method (HAM), is applied for predicting the temperature distribution in a porous fin with temperature dependent internal heat generation. The heat transfer through porous media is simulated using passage velocity from the Darcy's model. It has been attempted to show the capabilities and wide-range applications of the Homotopy Analysis Method in comparison with a type of numerical analysis as Boundary Value Problem (BVP) in solving this problem. The results show that the HAM is an attractive method in solving this problem.

**Keywords**: Homotopy analysis method; Porous fin; Temperature-dependent heat generation

## Introduction

Fins are frequently used in many heat transfer applications to improve performance. In the other hand, for many years, High rate of heat transfer with reduced size and cost of fins are main targets for a number of engineering applications such as heat exchangers, economizers, super heaters, conventional furnaces, gas turbines, etc. Some engineering applications such as airplane and motorcycle also require lighter fin with higher rate of heat transfer. Increasing the heat transfer mainly depend on heat transfer coefficient (h), surface area available and the temperature difference between surface and surrounding fluid. However, this requirement is often justified by the high cost of the high-thermal-conductivity metals, that cost of high thermal conductivity metals is also high. fin is porous to allow the flow of infiltrate through it. Extensive research has been done in this area and many references are available especially for heat transfer in porous fins. Described below are a few papers relevant to the study described herein. The theoretical study of MHD has been a subject of great interest due to its widespread applications, such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. For instance, MHD induced in rockets can improve heat transfer through porous fins, located on rocket surface. On the effect of MHD flow, although there are many studies regarding the free convection regime, there are only a few regarding the mixed convection regime. Chamkha et al. [1] studied the effects of localized heating (cooling), suction (injection), buoyancy forces, and magnetic field for the mixed convection flow on a heated vertical plate. Aldoss et al. [2] investigated the effect of MHD on heat transfer from a circular cylinder. Nonlinear problems and phenomena play an important role in applied mathematics, physics, engineering and other branches of science specially some heat transfer equations. Except for a limited number of these problems, most of them do not have precise analytical solutions. Therefore, these nonlinear equations should be solved using approximation methods. Perturbation techniques are too strongly dependent upon the so-called "small parameters" [3]. Other many different methods have introduced to solve nonlinear equation such as the δ-expansion method [4], Adomian's decomposition method [4], Homotopy Perturbation Method (HPM) [5-11] and Variational Iteration Method (VIM) [12-21]. Homotopy analysis method is another techniques), which was introduced by Liao. This method has been successfully applied to solve many types of nonlinear problem [22-28].

In this work, we have applied Homotopy Analysis Method to find the approximate solutions of nonlinear differential equations governing on porous fin with temperature dependent internal heat generation. Results demonstrate that HAM is simple and accuracy compared with the BVP as a numerical method. Also, it is found that this method is powerful mathematical tools and that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering.

## Analysis

As shown in Figure 1, a rectangular porous fin profile is considered. The dimensions of this fin are length L, width wand thickness t. The cross section area of the fin is constant and the fin has temperature-dependent internal heat generation. Also, the heat loss from the tip of the fin compared with the top and bottom surfaces of the fin is assumed to be negligible. Since the transverse Biot number should be small for the fin to be effective [29], the temperature variation in the transverse direction are neglected. Thus heat conduction is assumed to occur solely in the longitudinal direction [30]. Energy balance can be written as:

$$q(x) - q(x+\Delta x) + q^*.A\,\Delta x = \dot{m}\,c_p\left[T(x) - T_\infty\right] + h(p.\Delta x)\left[T(x) - T_\infty\right] \tag{1}$$

The mass flow rate of the fluid passing through the porous material can be written as:

$$\dot{m} = \rho\,\overline{\vartheta_w}\,\Delta x\,w \tag{2}$$

The value of $\overline{\vartheta}_w$ should be estimated from the consideration of the flow in the porous medium. From the Darcy's model we have:

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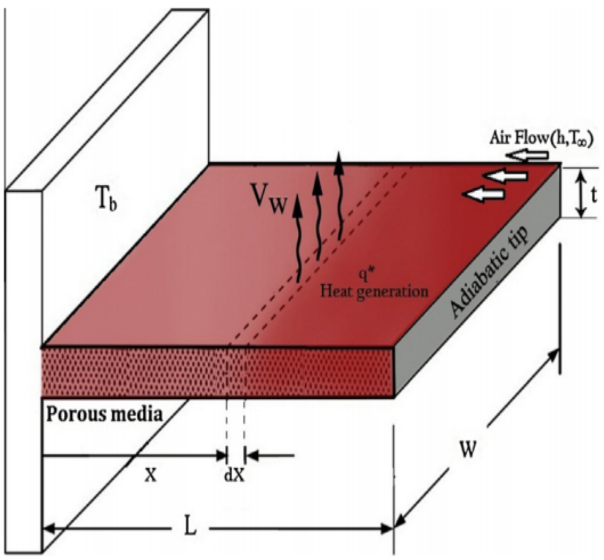


**Figure 1:** Schematic diagram for the problem under consideration.

$$\overline{\vartheta_w} = \frac{gk\beta}{\upsilon}\left[T(x)\text{-}T\infty\right] \tag{3}$$

Substitutions of Eqs. (2) and (3)into Eq. (1) yields:

$$\frac{q(x)-q(x+\Delta x)}{\Delta x}+q^*.A=\frac{\rho c_p g k\beta w}{\Delta x}\left[T(x)-T_\infty\right]^2+hp\left[T(x)-T_\infty\right] \tag{4}$$

As, $\Delta x \to 0$ Eq.(4)becomes

$$\frac{dq}{dx}+q^*.A=\frac{\rho c_p g k\beta w}{\Delta x}\left[T(x)-T_\infty\right]^2+hp\left[T(x)-T_\infty\right] \tag{5}$$

Also from Fourier's Law of conduction: $q=-k_{eff}\,A\frac{dT}{dx}$ (6)

Where $A$ is the cross-sectional area of the fin $A=w.t$ and $k_{eff}$ is the effective thermal conductivity of the porous fin that can be obtained from following equation:

$$k_{eff}=\varphi.k_f+(1-\varphi)k_s \tag{7}$$

Where $\varphi$ is the porosity of the porous fin. Substitution Eq. (6) into Eq. (5) leads to:

$$\frac{d^2T}{dx^2}-\frac{\rho c_p g k\beta w}{t\,k_{eff}\upsilon}\left[T(x)-T_\infty\right]^2+\frac{h\,p}{k_{eff}\,A}\left[T(x)-T_\infty\right]+\frac{q^*}{k_{eff}}=0 \tag{8}$$

It is assumed that heat generation in the fin varies with temperature as Eq.(9) [15]:

$$q^*=q_\infty^*\left[1+\varepsilon(T-T_\infty)\right] \tag{9}$$

Where $q_\infty^*$ is the internal heat generation at temperature $T_\infty$. For simplifying the above equations some dimensionless parameters are introduced as follows:

$$\theta=\frac{(T-T_\infty)}{(T_b-T_\infty)},\quad X=\frac{x}{L},\quad M^2=\frac{hpL^2}{k_0A},\quad Sh=\frac{Da\,x\,Ra}{kr}\left(\frac{L}{t}\right)^2$$
$$G=\frac{q_\infty^*}{h\,p(T_b-T_\infty)},\quad \varepsilon g=\varepsilon(T_b-T_\infty) \tag{10}$$

Where $Sh$ is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect so higher value of $Sh$ indicates higher permeability of the porous medium or higher buoyancy forces. $M$ is a convection parameter that indicates the effect of surface connecting of the fin. Finally, Eq.(8) can be rewritten as:

$$\frac{d^2\theta}{dX^2}-M^2\theta+M^2G(1+\varepsilon g\,\theta)-Sh\,\theta^2=0 \tag{11}$$

In this research we study finite-length fin with insulated tip. For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip and boundary condition will be,

$$\theta(1)=1\quad,\quad \theta'(0)=0 \tag{12}$$

## Homotopy Analysis Method (HAM)

For HAM solutions, we choose the initial guess and auxiliary linear operator in the following form:

$$\theta_0(x)=1 \tag{13}$$

$$L(\theta)=\theta'' \tag{14}$$

$$L(c_1x+c_2)=0 \tag{15}$$

Where $c_i(i=1,2)$ are constants. Let $\mathrm{P}\in[0,1]$ denotes the embedding parameter and $\eta$ indicates non–zero auxiliary parameters. We then construct the following equations:

**Zeroth–order deformation equations**

$$(1-P)L[\theta(x,p)-\theta_0(x)]=p\hbar H(x)N[\theta(x,p)] \tag{16}$$

$$\theta(0;p)=1;\qquad \theta'(1;p)=0 \tag{17}$$

For p=0 and p=1 we have: $\theta(x;0)=\theta_0(x)\quad \theta(x;1)=\theta(x)$

When $p$ increases from 0 to 1 then θ(x;p) varies from $\theta_0(x)$ to $\theta(x)$. By Taylor's theorem and using Eqs. (19), θ(x;p) can be expanded in a power series of p as follows:

$$\theta(x;p)=\theta_0(x)+\sum_{m-1}^{\infty}\theta_m(x)p^m\quad,\quad \theta_m(x)=\frac{1}{m!}\frac{\partial^m(\theta(x;p))}{\partial p^m}\bigg|_{p=0} \tag{20}$$

In which η is chosen in such a way that this series is convergent at p=1; therefore we have through Eq. (20) that,

$$\theta(x)=\theta_0(x)+\sum_{m-1}^{\infty}\theta_m(x), \tag{21}$$

**mth –order deformation equations**

$$L\left[\theta_n(X)-\chi_n\,\theta_{n-1}(X)\right]=\hbar\,H(X)R_n(X) \tag{22}$$

$$\theta(0;p)=0;\qquad \theta'(1;p)=0 \tag{23}$$

Where

$$R_n(X)==\frac{d^2\theta(X;p)}{dX^2}-M^2\theta_{n-1}+M^2G(1+\varepsilon g\;\theta_{n-1})-Sh\sum_{k=0}^{n-1}\theta_{n-1-k}\,\theta_k=0 \tag{24}$$

Now we determine the convergence of the result, the differential equation, and the auxiliary function according to the solution expression. So let us assume:

$$H(X)=1 \tag{25}$$

We have found the answer by maple analytic solution device. For three deformation of the solution are presented below

$$\theta_1(X)=\frac{1}{2}\hbar\left(-Sh-M^2+M^2G(1+\varepsilon g)\right)X^2+\frac{1}{2}\hbar Sh+\frac{1}{2}\hbar M^2-\frac{1}{2}\hbar M^2G -\frac{1}{2}\hbar M^2G\,\varepsilon g \tag{26}$$

$$\begin{aligned}\theta_2(X)&=0.041667\hbar^2X^4\left(-Sh-M^2+M^2G+M^2G\varepsilon g\right)\left(-2Sh-M^2+M^2G\varepsilon g\right)\\&+0.5\big[-\hbar Sh-\hbar M^2+\hbar M^2A+1.5Sh\hbar^2M^2A\varepsilon g-Sh\hbar^2-\hbar^2M^2-\hbar^2Sh^2-0.5M^4\hbar^2\\&+\hbar^2M^2A-1.5Sh\hbar^2M^2+0.5M^4\hbar^2A+Sh\hbar^2M^2G+M^4\hbar^2A\,\varepsilon g+\hbar M^2G\,\varepsilon g\\&-0.5M^4A^2\,\varepsilon g\hbar^2-0.5M^4G^2\,\varepsilon g^2\hbar^2+\hbar^2M^2G\,\varepsilon g\,\big]X^2+0.20833M^4h^2\\&-0.5\hbar^2M^2G+0.625Sh\hbar^2M^2-0.20833M^4\hbar^2G+0.5\hbar M^2+0.20833M^4G^2\varepsilon g^2\hbar^2\\&-0.41667SH\hbar^2M^2G-0.41667M^4\hbar^2G\varepsilon g+0.20833M^4G^2\varepsilon g\hbar^2-0.5\hbar^2M^2G\varepsilon g\\&-0.5\hbar M^2G\varepsilon g-0.5\hbar M^2G-0.62500Sh\hbar^2M^2G\,\varepsilon g+0.5Sh\,\hbar^2+0.5\,\hbar^2M^2\\&+0.5\,\hbar Sh+0.41667\hbar^2Sh^2\end{aligned} \tag{27}$$

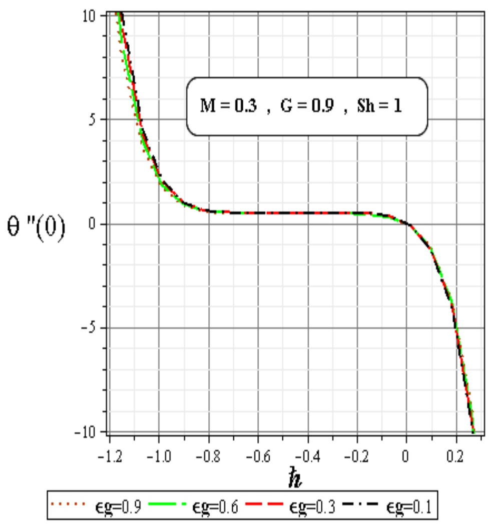


**Figure 2:** The $\hbar$ -validity for *X*=0 and different value of *H*.

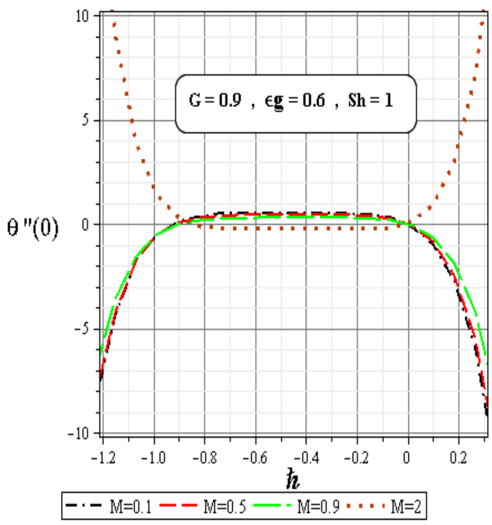


**Figure 3:** The $\hbar$ - validity for *X*=0 and different value of *M*.

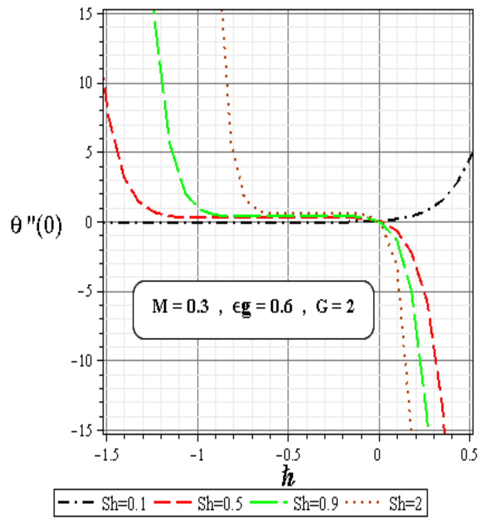


**Figure 4:** The h - validity for *X*=0 and different value of *Sh*.

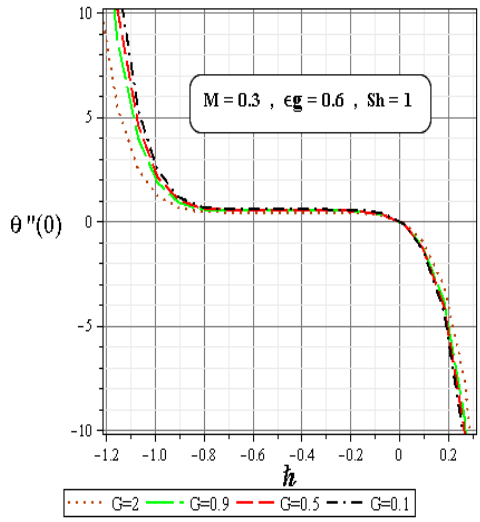


**Figure 5:** The $\hbar$ - validity for X=0 and different value of*G*.

The solutions $\theta(x)$ were too long to be mentioned here, therefore, they are shown graphically

## Convergence of the HAM solution

The series solutions contain the auxiliary parameter h. The validity of the method is based on such an assumption that the series (20) converges at p=1. It is the auxiliary parameter h which ensures that this assumption can be satisfied. As pointed out by Liao [23], in general, by means of the so-called h-curve, it is straightforward to choose a proper value of $\hbar$ which ensures that the solution series is convergent. In this way, we choose h=-0.2 in following computational works (Figures 2-5).

## Results and Discussion

In this manuscript, the Homotopy Analysis Method such as

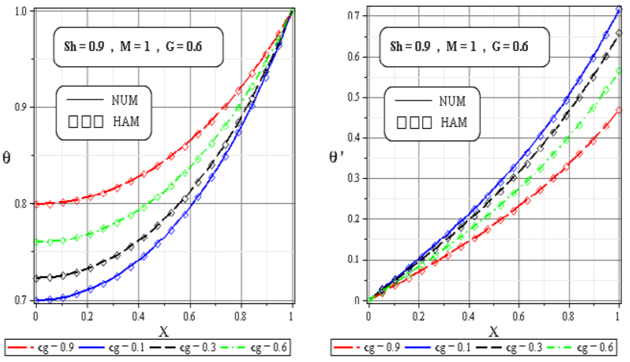


**Figure 6:** The comparison between the Numerical and HAM solution for $\theta$ and $\theta'$ atdifferent value of $\varepsilon g$ .

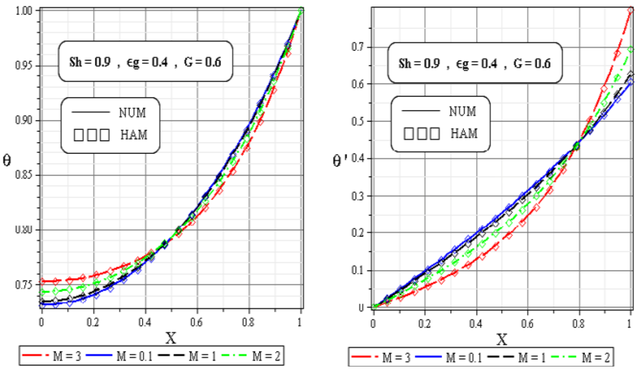


**Figure 8:** The comparison between the Numerical and HAM solution for $\theta$ and $\theta'$ atdifferent value of *M*.

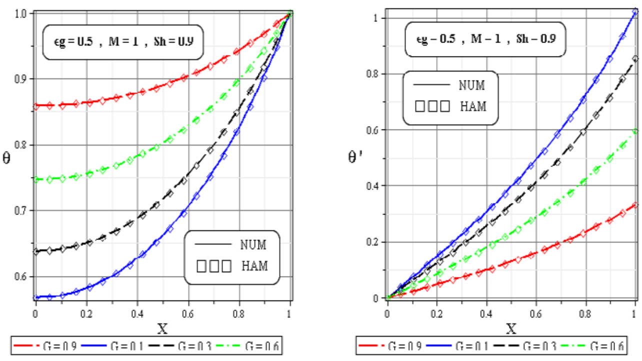


**Figure 7:** The comparison between the Numerical and HAM solution for $\theta$ and $\theta'$ atdifferent value of *G*.

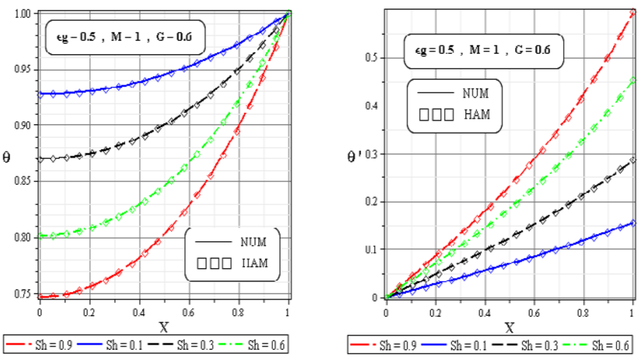


**Figure 9:** The comparison between the Numerical and HAM solution for $\theta$ and $\theta'$ atdifferent value of *Sh*.

| $X$ | $\theta(X)$ | | | $\theta'(X)$ | | |
|---|---|---|---|---|---|---|
| | **HAM** | **NUM** | **Error** | **HAM** | **NUM** | **Error** |
| 0.00 | 0.452726862 | 0.452787880 | 0.0000610676 | 0.000000000 | 0.000000000 | 0.000000000 |
| 0.05 | 0.453655038 | 0.453715480 | 0.0000604418 | 0.037163422 | 0.037139216 | 0.000024205 |
| 0.10 | 0.456450499 | 0.456509076 | 0.0000585769 | 0.074764710 | 0.0747149290 | 0.000049781 |
| 0.15 | 0.461146198 | 0.461201593 | 0.0000553947 | 0.113247727 | 0.113169914 | 0.000077812 |
| 0.20 | 0.467797557 | 0.467848300 | 0.0000507434 | 0.153068448 | 0.152959108 | 0.00010934 |
| 0.25 | 0.476483231 | 0.476527631 | 0.0000443994 | 0.194701325 | 0.194556142 | 0.000145183 |
| 0.30 | 0.487306202 | 0.487342342 | 0.000036140 | 0.238646034 | 0.238460303 | 0.00018573 |
| 0.35 | 0.500395212 | 0.500420955 | 0.0000257432 | 0.285434739 | 0.285203912 | 0.000230827 |
| 0.40 | 0.515906592 | 0.515919591 | 0.0000129992 | 0.335640062 | 0.335360431 | 0.000279631 |
| 0.45 | 0.534026494 | 0.534024250 | 0.0000022448 | 0.389883917 | 0.389553555 | 0.000330363 |
| 0.50 | 0.554973606 | 0.554953590 | 0.0000200162 | 0.448847455 | 0.448467480 | 0.000379975 |
| 0.55 | 0.579002383 | 0.578962239 | 0.0000401438 | 0.513282354 | 0.512858616 | 0.000423738 |
| 0.60 | 0.606406896 | 0.606344722 | 0.0000621731 | 0.584023786 | 0.583569072 | 0.000454714 |
| 0.65 | 0.637525367 | 0.637440131 | 0.0000852359 | 0.662005412 | 0.661542398 | 0.000463014 |
| 0.70 | 0.672745525 | 0.672637657 | 0.000107868 | 0.748276866 | 0.747842077 | 0.000434789 |
| 0.75 | 0.712510897 | 0.712383109 | 0.000127788 | 0.844024269 | 0.843673377 | 0.000350892 |
| 0.80 | 0.757328227 | 0.757186633 | 0.000141594 | 0.950594447 | 0.950409414 | 0.000185033 |
| 0.85 | 0.807776202 | 0.807631870 | 0.000144332 | 1.069523667 | 1.069622444 | 0.000098777 |
| 0.90 | 0.864515764 | 0.864386826 | 0.000128938 | 1.202571916 | 1.203121649 | 0.000549733 |
| 0.95 | 0.928302283 | 0.928216816 | 0.0000854667 | 1.351763967 | 1.352999072 | 0.001235105 |
| 1.00 | 1.000000000 | 1.000000000 | 0.000000000 | 1.519438801 | 1.521685895 | 0.002247094 |

**Table 1:** The results of HAM and Numerical methods for $\theta(X)$ and $\theta'(X)$ for $M = 2,\ \varepsilon g = 0.1, Sh = 0.9\ \ and\ \ G = 0.3$ .

analytical technique is employed to find an analytical solution of the temperature distribution in a porous fin. Figure 6 show comparison between the numerical solution and HAM solution for $\theta$ and $\theta'$ when *Sh*=0.9, *M*=1, *G*=0.6 and different value of *εg*.

Figure 7 illustrate the accuracy of HAM solution compare to numerical solution when, *εg*=0.5, *M*=1, *Sh*=0.9 and different value of G. Figure 8 show comparison between numerical solution and HAM solution when *εg*=0.4, G=0.6, *Sh*=0.9 and different value of *M*.

Figure 9 show comparison between numerical solution and HAM solution when *M*=1, G=0.6, *εg*=0.4 and different value of *Sh*. According to Table 1 and Figures 6-9, clearly show that the results by HAM are in excellent agreement with the exact solutions. Also, the auxiliary parameter h provides us with a convenient way to adjust and control the convergence and its rate for the solutions series.

## Conclusion

In this study, the MHD on a Porous Fin to a Vertical Isothermal Surface with temperature dependent internal heat generation was analyzed using HAM. The found that the approximations obtained by HAM are valid when compared with the exact solutions. It should be emphasized that the HAM provides us with a convenient way to control the convergence of approximation series. Finally, it has been attempted to show the capabilities and wide-range applications of the HAM in engineering.

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